STOCHASTIC GENERATION OF POINT RAINFALL DATA AT SUBDAILY TIMESCALES: A COMPARISON OF DRIP AND NSRP

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Stochastic Generation of Point Rainfall Data at Subdaily Timescales : A Comparison of DRIP and NSRP.

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Preface

One of the goals of the Climate Variability Program in the Cooperative Research Centre (CRC) for Catchment Hydrology is to develop and test computer programs for generating stochastic climate data at timescales from less than one hour to one year and for point sites to large catchments. The appropriate models will be part of SCL (Stochastic Climate Library – a suite of stochastic climate data generation models), a "product" in the CRC's Modelling Toolkit (see www.toolkit.net.au/scl).

This report describes the evaluation of two point subdaily stochastic rainfall models - the Newman-Scott Rectangular Pulse (NSRP) and the Disaggregated Rectangular Intensity Pulse (DRIP). The models are evaluated using relatively long pluviograph data from ten major Australian cities and regional centres.

Francis Chiew Program Leader – Climate Variability Program CRC for Catchment Hydrology

Summary

Two models for stochastic generation of point rainfall data at subdaily timescales are compared in this report: the Disaggregated Rectangular Intensity Pulse (DRIP) model of Heneker *et al.*, (2001) and the single site version of the Neyman-Scott Rectangular Pulse (NSRP) process model of Cowpertwait *et al.*, (2002). These two models are quite different in their conceptualisation of the rainfall process, but have both previously shown good reproduction of statistics not used in calibration – particularly Intensity-Frequency-Duration (IFD) curves.

The purpose of this study is to evaluate and compare the two models for use in the CRC for Catchment Hydrology's modelling toolkit. The two models were calibrated to ten major Australian cities/regional centres with relatively long pluviograph data.

The models were evaluated on their ability to reproduce 'standard' and extreme rainfall model statistics derived from the pluviograph record over a range of timescales (1, 6 and 24 hr) along with other daily, monthly and annual statistics derived from the longer daily rainfall series. A wide range of statistics are presented on a site-by-site basis allowing the user to determine if either model is applicable to a given project.

The results indicate that both models reproduce satisfactorily most of the rainfall statistics of the historical data. The NSRP model reproduces most statistics well, but performs poorly in regard to some wetspell and dryspell statistics. The DRIP model also reproduces most statistics well, but has problems simulating the short duration IFDs. Both models are recommended for use in the CRC for Catchment Hydrology's modelling toolkit, with the user advised to check the rainfall statistics (in particular those mentioned above) to ensure that they are adequate for a given project.

Go to: www.catchment.crc.org.au/pdfs/technical200409_C.pdf

Appendix D: NSRP Site-by-Site Statistics

Go to: www.catchment.crc.org.au/pdfs/technical200409_D.pdf

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1. Introduction

1.1 Why Stochastically Model Subdaily Point Rainfall?

Due to the complex and chaotic nature of climate, rainfall is often modelled as a stochastic process. The purpose of such modelling is to produce replicated simulated series of data, which are representative of the range of scenarios that could possibly occur. These simulated series can be used in design in the place of long series of observed data. As there is typically less than 15 years of observed subdaily rainfall at sites throughout Australia, stochastic rainfall models are an important tool in providing input rainfall data for design that requires point rainfall data as an input.

Examples of occasions where point rainfall generation (at the subdaily timescale) may be important are especially evident in urban hydrology – particularly in relation to flood estimation (see Kuczera *et al.*, 2003 for a discussion). Coombes *et al.*, (2003) use rainfall generated by a point subdaily rainfall model in testing the implications of using household rainwater tanks for household water use on household demand and household runoff. Cowpertwait *et al.*, (2002) apply a spatial-temporal subdaily rainfall model to hourly data from the Arno catchment, Italy – the resulting simulations being used in flood studies.

Stochastic models provide a relatively simple method of representing the complex rainfall processes occurring. The alternative to stochastic modelling is to use deterministic equations based on physical processes, as is done over large spatial grid scales in general circulation models (GCM's). However, this spatial scale (typically \approx 100 *km* x 100 *km*) is too large for many hydrologic design applications. Progress is being made in downscaling the large spatial scale deterministic simulations to produce stochastic rainfall series over a smaller lattice of points or gridded area (e.g. Charles *et al.*, 1999, Venugopal *et al.*, 1999, Bellone *et al.*, 2000). Yet the computation time involved for multiple generated series of GCM's is currently prohibitive (given the design lifetime of most hydrological applications), and are sensitive to starting conditions. Alternatively, stochastic rainfall generated at a larger timescale can be disaggregated to

a smaller timescale (eg. Sivakumar *et al.*, 2001, Koutsoyiannis *et al.*, 2003). The strength of the parametric point stochastic approach is its simplicity, and ability to subjectively (through choice of distributional form) limit the extremes that may be produced.

Spatial generation of stochastic rainfall at subdaily timescales over a series of sites has been performed by a number of authors (Northrop, 1998, Cowpertwait *et al.*, 2002). It is the purpose of this study however to test single-site subdaily rainfall models for inclusion into the CRC for Catchment Hydrology's modelling toolkit.

This study selects two of the better performing point subdaily rainfall models, and applies them to 10 Australian capital cities/regional centres. These 10 sites were chosen due to the relative availability of long rainfall records recorded at the 6-min timescale. The application of the DRIP and NSRP models demonstrates the current versions of each model to Australian conditions.

1.2 Previous Attempts at Point Stochastic Subdaily Rainfall Generation

Stochastic modelling has been an area of active research for some years now, and research has generally focussed on two approaches: cluster based and event based models.

Also known as 'alternating renewal' or 'profile based' models, *event based models* break the conceptual rainfall process into a series of events characterised by inter-arrival time, storm duration and mean storm intensity (Eagleson, 1978, Koutsoyiannis and Pachakis, 1996, Menabde and Sivapalan, 2000, Heneker *et al.*, 2001) – see Figure 1(a). A storm is identified by the occurrence of a dry period (rainfall less than a threshold value, typically zero) that is longer than some specified duration ranging typically from 2-9 hours. These models use various methods to disaggregate a simulated storm event to the desired timescale. Some models disaggregate via a nondimensionalised storm profiling/scaling (Woolhiser and Osborn, 1985, Koutsoyiannis and Foufoula-Georgiou, 1993, Koutsoyiannis and Pachakis, 1996, Heneker *et al.*, 2001, Koutsoyiannis and Mamassis, 2001). Others use multi-fractal disaggregation techniques (Menabde and Sivapalan, 2000).

Cluster based models conceptualise rainfall as a series of storm arrivals, with rainfall cells associated with each storm (these rainfall cells usually have a random duration and intensity) such that the total intensity at any time is the sum of the intensities of all cells active at that time – see Figure 4 for an example. Considerable research has been focussed on cluster based modelling, specifically Neyman-Scott (NS) and Bartlett-Lewis (BL) type cluster processes (Rodriguez-Iturbe *et al.*, 1987, Cowpertwait, 1991, Onof and Wheater, 1993, Cowpertwait and O'Connell, 1997). These models differ in the displacement of cell origins relative to storm origins, and there have been several empirical studies comparing the two models (eg. Velghe *et al.*, 1994). Cowpertwait (1998) has shown analytically that BL and NS rectangular pulse rainfall models were equivalent up to second-order properties.

In terms of application to Australian conditions within the literature, DRIP has been the most widely applied event based model (Heneker, 2002), with Menabde and Sivapalan (2000) applying their model to a single site only. DRIP was found within the study of Heneker *et al.* (2001) to reproduce aggregation and IFD statistics not used in calibration well, and hence was chosen for evaluation. Of the cluster based models, the BL rectangular pulses model and subsequent generalisations has been used most widely (Gyasi-Agyei and Willgoose, 1997, Gyasi-Agyei, 1999, Gyasi-Agyei and Willgoose, 1999), and performed well in terms of reproduction of extreme and aggregation statistics. However, a later version of the Neyman-Scott rectangular pulse model was chosen for comparison. This was chosen for three reasons;

- the third order properties of the rainfall process have been derived by Cowpertwait (1998);
- this model has been generalised for future use in spatial modelling studies (Cowpertwait *et al.*, 2002) and;
- the model has been further generalised to allow the superposition of two or more NSRP processes (Cowpertwait, 2003 details of which will be provided within the model description).

2. DRIP

2.1 Model Description

Following the description given in Heneker (2001), Disaggregated Rectangular Intensity Pulse (DRIP) conceptualises rainfall as a series of storm and interstorm events – see Figure $1(a)$. In terms of the observed rainfall record, a storm is defined as any period of positive rainfall which is separated by a minimum dry period of two hours. There are three random event variables: the interstorm duration (t_a) , the storm duration (t_a) and the average storm intensity (*i*), with the storm depth (*d*) defined as the product of i and t_i . Thus the storm is considered a series of rectangular intensity pulses (DRIP). The second stage of the model determines the temporal distribution of rainfall within each event using a disaggregation scheme as shown in Figure 1(b).

2.2 DRIP Event Data Calibration

As DRIP breaks the pluviograph data into a series of events, a method of calibration called maximum

likelihood can be used to obtain model parameters for simulation. This method relies on formulation of the model likelihood. Consider a set of observations $X = \{x_i, i=1,...,n\}$, each considered as being random realisations *X* of some stochastic process. If these observations are considered statistically independent, we can write the likelihood (likelihood of observing the data), given a model *M* and associated parameter set θ_M as:

$$
f(\mathbf{X} | M, \boldsymbol{\theta}_M) = f(x_1, \dots, x_n | M, \boldsymbol{\theta}_M) = \prod_{i=1}^n f(x_i | M, \boldsymbol{\theta}_M)
$$
\n(1)

The final product term signifies the assumption of independence between events. The general likelihood shown in Equation 1 is used for all events within DRIP.

Interstorm and storm durations are considered independent within DRIP, and a mixture of the Generalised Pareto (GP) distribution and the Power Law is used for the cumulative distribution function $P_X(X \leq x \mid M, \theta_M)$ (see Lambert and Kuczera, 1998 for details):

$$
\ln\left[1-P_X\left(X\leq x\right|M,\theta_M\right)\right]=\frac{1}{\theta_1}\ln\left(1-\theta_1\frac{x}{\theta_2}\right)-\theta_3x^{\theta_4}\theta_1<0,\ \theta_2,\theta_3,\theta_4>0\tag{2}
$$

Figure 1. DRIP Model of Precipitation Event Series: (a) Generation of a Time Series of Rectangular Rainfall Pulses or Events; and (b) a Random Shaped Hyetograph Produced by the Disaggregation Scheme. Figure from Heneker (2001).

The parameter ranges shown above are required to ensure that the cumulative distribution function is monotonically increasing and hence provides a valid probability distribution.

The likelihood function Equation 1 is complicated by the fact that the exact durations (*x*) of the storm and interstorm events are not known. For example an apparent six bin storm extracted from a six minute pluviograph record could have in reality lasted anywhere between 24 and 36 minutes – as shown in Figure 2.

Figure 2. DRIP Binned Data Schematic (adapted from Lambert and Kuczera (1998)).

Lambert and Kuczera (1998) deal with this by using an approximation to the expected probability of the event duration lying between the two extremes (see Equation 3).

Here *s* and *e* represent the actual start and end times of the event respectively. These start and end values are integrated between the possible start and end times with lower and upper bounds signified by the subscripts *l* and *u*.

Equation 3 is substituted into Equation 1 and optimisation is performed A maximum likelihood parameter is estimated for the storm duration and interstorm events seperately. The optimisation method used in this study was the Shuffled Complex Evolution algorithm detailed in Duan *et al.*,(1992).

Maximum likelihood is again used for the storm intensities parameter estimation. The storm intensities are modelled as being conditionally dependent on storm duration – as detailed below. The GP distribution was used for the distribution function of storm intensities (see Equation 4).

Here *x* and t_d are the storm intensity and duration respectively. A broken line function ($\mu = fn(1n(t_a))$ and $\sigma = fn(1n(t_d)))$ is used to model the dependence on duration to the mean $(μ)$ and the standard deviation $(σ)$ of this GP distribution – see Figure 3 for a schematic. These functions describe the relationship of mean and standard deviation of intensity with log storm duration, for the mean (see Equation 5).

$$
f(X|M, \theta_{M}) = P(X = e - s : s \in (s_{i}, s_{u}), e \in (e_{i}, e_{u}) | M, \theta_{M})
$$

\n
$$
= \frac{1}{s_{u} - s_{i}} \int_{s_{u}}^{s_{u}} P(X = e - s : e \in (e_{i}, e_{u}) | s, M, \theta_{M}) ds
$$

\n
$$
= \frac{1}{s_{u} - s_{i}} \left[\int_{s_{i}}^{s_{u}} P_{X}(X \le e_{u} - s | M, \theta_{M}) ds - \int_{s_{i}}^{s_{u}} P_{X}(X \le e_{i} - s | s, M, \theta_{M}) ds \right]
$$

\n
$$
\approx P_{X} \left(X \le e_{u} - \frac{s_{u} + s_{i}}{2} | M, \theta_{M} \right) - P_{X} \left(X \le e_{i} - \frac{s_{u} + s_{i}}{2} | M, \theta_{M} \right)
$$

\n(3)

$$
\ln\left[1 - P_X\left(X \le x \mid t_d, M, \boldsymbol{\theta}_M\right)\right] = \frac{1}{\theta_1} \ln\left[1 - \theta_1 \frac{x}{\theta_2}\right] \quad \theta_1 < 0, \ \theta_2 > 0 \tag{4}
$$

$$
\mu = fn\left(\ln(t_d)\right) \begin{cases} = \mu_1 & \text{if } t_d \le b_1 \\ = \mu_{b_k} + \left(\ln(t_d) - \ln(b_k)\right) \left(\frac{\mu_{b_k+1} - \mu_{b_k}}{\ln(b_{k+1}) - \ln(b_k)}\right) & \text{if } b_k < t_d \le b_{k+1} : k = 2, ..., nb - 1 \quad (5) \\ = \mu_{b_{ab}} & \text{if } t_d > b_{nb} \end{cases}
$$

where $\{b_k, k = 1, \ldots, nb\}$ are the set of breakpoint locations. An identical function is used for the standard deviation of intensity σ . μ_h and σ_h parameters are fitted at each of the breakpoint locations, and related back to the GP parameters through $\theta_1 = \mu((\mu^2 / \sigma^2) - 1)/2$ and $\theta_2 = \mu((\mu^2 / \sigma^2) + 1)/2$.

DRIP has been altered since the study of Heneker *et al.*, (2001) in an attempt to reduce the subjectivity of choosing the number of breakpoints and their associated durations – such that the model could be applied by a user with little experience. Within the Heneker *et al.*, (2001) study, different numbers of breakpoint were used at each site, with differing locations also. Nine standard breakpoint locations were used {0.2, 0.6, 1.1, 2.0, 3.0, 6.0, 10.0, 15.0, 20.0} (hrs) here. The intention with these breakpoints (compared to the 5-8 as published in previous studies)

was that this range should cope with the complexities shown in Australian rainfall based on previous experience. The disadvantage of using so many breakpoints is that there are more parameters to fit (9 breakpoints = 9 mean parameters $+$ 9 standard deviation parameters). If the model is calibrated on a monthly basis, apart from leading to a relatively large number of parameters overall, there is occasionally no data between each of the breakpoints to fit to. This allows the parameters to take any value without having any influence on the likelihood. This can have potentially disastrous results for simulation. To combat the free reign of these parameters, bounds are set for the optimisation through an initial fit using the function in Equation 6.

$$
\mu = f n_{\mu} \left(\ln(t_{d}) \right) \begin{cases} = \mu_{b_{1}} & \text{if } t_{d} \leq b_{1} \\ = \mu_{b_{k}} + \text{frac} \left(\mu_{b_{k}+1} - \mu_{b_{k}} \right) - \gamma_{\mu} \text{frac} \left(1 - \text{frac} \right)^{\beta_{\mu}} & \text{if } b_{1} < t_{d} \leq b_{nb} \\ = \mu_{b_{ab}} & \text{if } t_{d} > b_{nb} \end{cases}
$$
\nwhere\n
$$
\text{trace} = \frac{\ln(t_{d}) - \ln(b_{nb})}{\ln(b_{nb}) - \ln(b_{1})}
$$
\n(6)

Figure 3. DRIP Duration-Intensity Broken Line Functions for Mean and Standard Deviation. Example Shows Four Breakpoints - with Mean and Standard being Constant Outside the First and Last Breakpoint.

This is essentially the previous broken line relationship (between the first and last breakpoint) with the addition of an extra term. This extra term has the functional form of the Beta density without a normalising constant. This relationship will be called the beta calibration from herein, while the 9-point broken line function will be called the broken line. The beta calibration reduces the number of parameters used – from 18 to 10 for each month. The beta calibration was trialled in place of the broken line calibration. However, the beta calibration was found to be less flexible in fitting all Australian conditions (Lambert and Kuczera, 2003, *pers. comm.*). However, it was used as a quick method of bounding the search region for the broken line fit. After the beta fit was completed, bounds are set on the broken line search according to:

$$
lbm (b_k) = \max \{ 0.01, 0.05 \hat{\mu}_{b_k} \}
$$

\n
$$
ubnd (b_k) = \min \{ \max \{ \hat{\mu}_{b_k} + 0.1, 1.0, 0.95 \hat{\mu}_{b_k} \}, 20 \}
$$
\n(7)

The $\hat{\mu}_{b_n}$ signifies the maximum likelihood estimates at each of the breakpoints from the beta function calibration.

Now returning to the intensity likelihood, there are two different functions used for evaluation of $f(x_i|M,\theta_M)$ (see Equation 8).

Here p_x is the chosen continuous density function representing the distribution of the random variable *X*, and P_x represents the cumulative probability that the censored value is less than a particular cut-off (minInten). In simple terms, we use the known value (density) function when we know the measurements value, else we use the probability of it being below a particular value. Although not mentioned in previous DRIP studies, this censoring is required due to data sampling effects – see DRIP storm duration histograms in Appendix C (eg. Figure C.1(a)). Due to the minimum gradation of the pluvio records (0.01 mm), there is a large occurrence of 0.1 mm/hr values

(0.01 mm/0.1 hr). Hence minInten is set to 0.1 mm/hr within calibration.

This censoring does not recognise that other intensity values over occurrence due to the pluvio gradation (eg. 0.2 mm/hr). This could be formally addressed, as each storm duration has a minimum intensity that can occur $(0.1 \text{ mm/hr}$ for $t_d=6 \text{ min}$ storms, 0.02 mm/t_d for 6min> t_d ≥2*hrs*, 0.03mm/ t_d for 2*hrs*> t_d ≥4*hrs* and so on). That is, the minInten value could be considered a function of storm duration.

It is noted here that the treatment of intensities is not as rigorous as that of interstorm and storm durations. Here, the storm durations used to calculate the intensities are considered as known values – which in turn results in the assumption that the intensity values are known exactly. However, as recognised in the storm duration fitting, the exact duration of the storm is not known. This in turn means the intensity value is not known exactly. An evaluation of the expected probability of the intensity lying between two values should be calculated. This is complicated by the fact that the intensity is conditioned on storm duration. Again it is treated as a known value, where it should ideally be treated as lying somewhere between two bounds.

2.3 DRIP Event Disaggregation

Once DRIP simulates a storm duration and intensity, the event is broken down to reproduce the short timescale temporal characteristics of the data using a conditional random walk on a dimensionless mass curve. This mass curve, with the non-dimensionalised storm duration $\tau = t/t_d$ on the x-axis (where *t* is the time since the start of the event $0 < t < t_d$), and $\delta = d(t) / d(t_a)$ where $d(t)$ is the cumulative rainfall up until time *t*. This curve is considered a stochastic process also, hence a random walk is used to generate it. The storm is broken into intervals, and an associated depth jump is determined for each interval (according to a lognormal jumping distribution). Although quite important in IFD curves, and

$$
f(X = x_i \mid M, \theta_M) = \frac{P_X(X \le \min \ln \ln n \mid M, \theta_M) \quad \text{if} \quad x_i \le \min \ln \ln n}{P_X(X = x_i \mid M, \theta_M) \quad \text{otherwise}}
$$
(8)

intrastorm temporal variability, disaggregation will not be discussed in detail. The focus of this study is on the greater aggregation scales, of the order of storm event durations. Further details of this disaggregation can be found within Heneker *et al.*, (2001).

It is noted however that the calibration of DRIP disaggregation parameters does not currently consider rainfall events with less than 1 hour of cumulative wet bins. This excludes many rainfall events from having an influence on disaggregation. If the assumption used in disaggregation - that the process can be nondimensionalised - is correct, ignoring these data will increase the sampling variability of the parameter estimates. If however the process is not similar for different storm durations/depths, rejecting these smaller duration storms will bias the parameters towards the longer duration storm characteristics. Refinement of the disaggregation method (and addressing this issue) is a current direction of research of the model authors.

3. NSRP

3.1 Model Description

The Neyman-Scott rectangular pulse (NSRP) cluster model is the single point version of the spatial NSRP model presented within Cowpertwait *et al.*, (2002). The NSRP model conceptualises rainfall as consisting of a series of storms, with an associated set of (rectangular pulse) rainfall cells with each storm – see Figure 4.

For a stationary period the model is summarised by the following random variables and model parameters:

- (i) the time *T* between adjacent storm origins is an independent exponential random variable with parameter λ (so the storm origins arrive according to a Poisson process);
- (ii) the waiting time W for a cell origin after a storm origin is an independent exponential random variable with parameter β;
- (iii) the lifetime *L* of a cell is an independent exponential random variable with parameter η;
- (iv) the number of cells *C* per storm is taken to be an independent random variable that remains constant throughout the cell lifetime *L*, and is taken to be a Weibull random variable so that $P_X(X > x) = e^{-(x/\theta)^{\alpha}}$.

In recent work, Cowpertwait (2003) further generalises the NSRP to allow the superposition of independent NSRP processes, giving the superposed NSRP process:

$$
SNSRP(n) = \sum_{i=1}^{n} NSRP_i
$$
 (9)

with parameter set:

$$
\theta_{\text{SNRSRP}} = {\lambda_1,...,\lambda_n,\mu_{C_1},...,\mu_{C_n},\beta_1,...,\beta_n,\eta_1,...,\eta_n,\alpha_1,...\alpha_n,\theta_1,...\theta_n}
$$

where $SNSRP(1) \equiv NSRP_I$ (the original NSRP) process). Although some attempts were made to calibrate the *SNSRP*(2), only results relating to the simpler *SNSRP*(1) are presented. This simpler model was chosen firstly because it has a simpler structure,

Figure 4. A Schematic of the Neyman-Scott Model (derived from Cowpertwait (1991)).

secondly it has had wide exposure in the literature and finally the results for the *SNSRP*(2) did not improve upon the *SNSRP*(1) results significantly (specifically regarding wet and dryspell statistics).

3.2 NSRP Calibration

As the definition of the NSRP allows overlapping of cells and storms, events are no longer defined. This in turn means that the derivation of a likelihood for the rainfall accumulation amounts is difficult (if not intractable). Therefore, a different form of calibration is used for such models, this method revolves around matching (as closely as possible) rainfall aggregation statistics at various timescales. Therefore, theoretical functions relating these aggregation statistics to parameter values are required. Although not presented here due to their complexity, equations for the NSRP rainfall mean $\mu(h)$ and variance $\sigma^2(h)$ (Rodriguez-Iturbe *et al.*, 1987), skew ξ(*h*) (Cowpertwait, 1998), lag one autocorrelation ρ(*h*), dry probability *PD*(*h*) (Cowpertwait, 1991), wet-to-wet φ*WW*(*h*) and dry-todry φ*DD*(*h*) transition probabilities (Cowpertwait *et al.*, 1996) for any interval length *h* have been derived. The statistical properties, at aggregation level *h*, of the superposed process *SNSRP*(*n*) (abbreviated below to *S_n*) are related to the equivalent properties of each NSRP process according to:

$$
\mu_{S_n}(h) = \mu_1(h) + ... + \mu_n(h) \tag{10}
$$

$$
\sigma_{S_n}^2(h) = \sigma_1^2(h) + \dots + \sigma_n^2(h)
$$
 (11)

$$
\gamma_{S_n}(h) = \gamma_1(h) + \dots + \gamma_n(h) \tag{12}
$$

$$
\xi_{S_n}(h) = \xi_1(h) + ... + \xi_n(h)
$$
 (13)

$$
PD_{S_n}(h) = PD_1(h) \times \ldots \times PD_n(h) \tag{14}
$$

$$
\phi_{S_n}^{DD}(h) = PD_{S_n}(2h) / PD_{S_n}(h)
$$
 (15)

$$
\phi_{S_n}^{WW}(h) = \left\{1 - 2PD_{S_n}(h) + PD_{S_n}(2h)\right\}/\left\{1 - PD_{S_n}(h)\right\}
$$
\n(16)

Other functions used to fit the model within this study are:

Coefficient of variation:

$$
\mathbf{v}_{S_n}\left(h\right) = \sigma_{S_n}\left(h\right) / \mu_{S_n}\left(h\right) \qquad (17)
$$

Autocorrelation (lag one):

$$
\rho_{S_n}(h) = \gamma_{S_n}(h) / \sigma_{S_n}^2(h) \qquad (18)
$$

Coefficient of skewness:

$$
\kappa_{S_n}\left(h\right) = \xi_{S_n}\left(h\right)/\sigma_{S_n}^3\left(h\right) \qquad (19)
$$

Apart from the hourly rainfall mean, the statistics

$$
\psi(h) \in \left\{ \nu_{S_n}\left(h\right), \rho_{S_n}\left(h\right), \kappa_{S_n}\left(h\right), PD_{S_n}\left(h\right), \phi_{S_n}^{WW}\left(h\right), \phi_{S_n}^{DD}\left(h\right) \right\}
$$

were used in fitting the NSRP model over three timescales of aggregation: 1, 6 and 24 hrs. Whereas in previous studies subsets of ψ(*h*) were used in calibration, the full set is used here for the given durations. All statistics were used in calibration to ensure that incorporation of one statistic in calibration did not have greatly negative affects on some other vital statistics. Data was pooled on a monthly basis, sample estimates $\hat{\psi}(h)$ were obtained for each of the statistics ψ(*h*).

For each calendar month the NSRP was fitted by minimising the following least squares function (see Equation 20).

The weights w_h used were arbitrarily set to being equal, as was done in the Cowpertwait *et al.*, (1996) study. Equation (20) is minimised using a bounded parameter space optimisation routine for parameters ${\lambda_{i,i}, \mu_{C_i}, \beta_{i,i}, \eta_{i,i}, \alpha_{i,i}: i = 1, ..., 12; j = 1, ..., n}$. The Weibull scale parameter $\{\theta_{i,j}: i = 1,...,12; j = 1,...n\}$ is estimated directly from the sample mean using the equation $\hat{\theta}_{j,i} = \hat{\mu}_i / \mu_{j,i}$, resulting in exact reproduction of the sample mean.

$$
SS(n) = \sum_{h=1,6,24} w_h \cdot \left\{ w_h \cdot \left\{ w_{(h)e_1^{\{v_{S_n}(h), v_{S_n}(h), w_{S_n}(h), p_{S_n}(h), \phi_{S_n}^{nW}(h), \phi_{S_n}^{D0}(h))}\right| \left(1 - \frac{\psi(h)}{\hat{\psi}(h)}\right)^2 + \left(1 - \frac{\hat{\psi}(h)}{\psi(h)}\right)^2 \right\}
$$
(20)

4. Data and Aggregation Statistics

Pluviograph data from ten major cities and regional centres were chosen for this study. These sites were chosen due to the relatively long length of records available, at least 45 years of data (with the exception of Adelaide). Other Bureau of Meteorology pluviograph sites (that have been digitised) throughout Australia have lengths typically in the range 15-25 years. Thus, this can be considered as being the best possible opportunity for identification of model parameters in Australian conditions. The details of the sites used are shown in Table 1. Exact start and end dates of records are shown, along with the percentage of records flagged as missing (-9999 in Bureau records), and percentage flagged as bad/corrupted (-8888 or negative accumulated value). The total percentage unused is simply the sum of the missing and bad values.

4.1 Handling of Missing Data

4.1.1 DRIP

One of the proclaimed benefits of the event based approach compared to the cluster based approach is that event based models are more robust in the presence of missing or corrupted data (Heneker *et al.*, 2001). Although not examined in detail in that study, this statement relies upon several points. This statement will be examined by looking at the two methods of extraction of calibration data.

There are four possible scenarios when extracting storm and interstorm events from the pluviograph record involving missing or corrupted data.

- 1. A -9999/-8888 value occurs within two hours (minimum interstorm duration) after a wet bin (greater than zero). As the storm may have been part of a larger storm DRIP discards such storms.
- 2. A -9999/-8888 value occurs later than two hours (minimum interstorm duration) after a wet bin (greater than zero). If -8888 DRIP keeps the interstorm event, else discards the interstorm event.
- 3. A -9999/negative accumulation value occurs within two hours (minimum interstorm duration) before a wet bin (greater than zero). As the storm may have been part of a larger storm DRIP discards

such storms.

4. A -9999/negative accumulation value occurs later than two hours (minimum interstorm duration) before a wet bin (greater than zero). If a negative accumulation value, DRIP keeps the interstorm event, else discards the interstorm.

Not previously stated in the studies involving DRIP, is the treatment of the interstorm events occurring before or after a -8888/negative accumulated value. If the pluviograph records are absolutely accurate, and the -8888 values correspond to times where it is actually raining, the interstorm events should be included in the record. However, inspection of the timing of these accumulation values shows that often complete days starting at 9 am are flagged with -8888. This would indicate that the precise timing of the events is not known (it is unlikely that so many storms started and ended at 9 am – the standard measurement time for daily raingauges). Hence, including these interstorm events introduces a negative bias, as the actual interstorm event durations could possibly be longer.

It might be expected that such biased interstorm events also be omitted. However, this is not the case within DRIP. The reason for inclusion of such interstorm events is that exclusion biases the data to a greater degree. Although not shown here, interstorm events preceding/following a -8888 value are significantly longer on average than the remaining interstorm events. The current method therefore accepts that there is bias in the interstorm events, yet as the record would have lost significantly long interstorm events through trashing, they are kept for calibration. A conjectured mechanism for the proportionately higher occurrence of such interstorm events is that the pluviograph is more likely to be corrupted after long interstorm events (through non-use, accumulation of dust etc). It is possible that such biases occur in interstorm events surrounding missing (-9999) values, however this was not tested in this report.

Similarly to the bias found in interstorm events starting/finishing with a -8888, it is possible that the storms that occur during the -8888 periods are also greater/or less than the remainder of the record. That is, these storms after long interstorm events may be greater on average in duration and intensity than the remainder of the storms. However, this is only a conjecture as the actual storm length is not known (if the 24 hr 9 am values are presented). Within the storm statistics presented in this report, a significant (although small ~ 0.1) correlation is found between interstorm event length and the following storm intensity. This is one piece of evidence suggesting that more intense storms may follow long interstorm events during a -8888 value. A possible method to test this would be to test if the 24 hr accumulation values differ from those for non-corrupted daily wet values. Alternatively, if the -8888 values are less than 2 hrs in duration (guaranteeing it being a singular storm event), these duration and intensity (from the 24 hr accumulation value) could be tested against other storms of the same duration.

Therefore, interstorm events are biased (shortened to some degree by inclusion of interstorm events starting/ending with -8888, and possibly biased by exclusion of interstorm events starting/ending with -9999). It is reasonable to suggest that the storms rejected during -8888 values are also biased (given the small correlations between interstorm event duration, storm intensity and storm duration).

4.1.2 NSRP

The NSRP uses aggregation statistics in calibration based on the entire record (with -8888 wet periods not being used) for calculation of statistics such as hourly mean, variance, skew or dry probability. Mean and variance will be biased downwards, while dry probability is biased upwards. Hours partly missing or corrupted are trashed. An alternative approach with the NSRP would be to reject entire months if there is a missing or corrupted value within. However, in this study it was considered too much information would be lost due to the regular occurrence of such events within the records used.

4.1.3 Discussion

It is not clear which of the two methods (event versus cluster) is superior in the presence of missing and/or corrupted data. The statement that event based models are more robust in the presence of missing or corrupted data (Heneker *et al.*, 2001) is questionable.

4.2 Data Details

Extraction of DRIP events was undertaken for the ten sites from the start date to the end date of the pluvio

record. Also shown in Table 1 are the start and end dates used for aggregation statistics calculations. As it is desired that the simulation length be equal to the aggregation statistics length with which they are compared (in validation), the pluvio records were clipped to use complete years. This ensures that the confidence limits produced through repeated simulation accurately represents the sampling variability of the model (i.e. longer simulation length produces smaller sampling variability for a given statistic for a stationary model). Hence, the number of years simulated is equal to the number of aggregation years minus the number of missing years.

These aggregation statistics derived from the pluviograph records were used for calibration of the NSRP. Although providing a slightly shorter record for calibration of NSRP compared to the non-clipped pluvio record for DRIP, it is expected such differences attributable to the extra data be minor (given that it is a small proportion of the overall data length). Moreover, as use of complete years is a current requirement of the NSRP calibration, where DRIP allows incomplete years within calibration, this is a test of each model's ability to use the available data.

It is noted that the majority of the years 1873-1876 for the Melbourne record are missing. Also, the percentage missing value presented includes the missing years as missing data (hence the high value for Melbourne).

At least one hundred replicates with the length of the aggregated series were produced. This replication is used as an indicator of sampling variability. For all validation and calibration statistics, the median simulated value is produced along with the associated 90% confidence limits. The presence of missing data within the aggregated records results in the simulated data length being longer than the true observed record, and resultant confidence bounds produced underestimating true sampling variability. As the total percentage missing or corrupted pluviograph records are around 5% (excluding completely missing years), this was judged not to have a significant impact on the conclusions made. Moreover, statements of the relative fit of DRIP and NSRP to the observed data can still be made as simulation lengths are identical for both models.

Table 2 presents the detail of the daily data used for

aggregation statistics. Situated at the same sites as the pluviographs, this daily data spans a longer period and is of much better quality than the pluviograph record at the same site (very little flagged as missing or corrupted). Hence, the daily data was used for daily, monthly and annual aggregation statistics. It is assumed for the purposes of comparison that the statistics remain stationary for the duration of the pluviograph and daily record. The simulations used for the pluviograph aggregation statistics comparisons were reused for the daily comparisons. One hundred

simulations of the length of the daily record were produced, hence there were more than 100 replicates for the sub-daily data (e.g. Adelaide: 100 daily replicates \approx 134 replicates $*$ 35 years pluviograph /47 years daily). Although longer periods of daily data were available at some of the sites (going further back in time), these other years were not used due to the presence of missing months/years of record.

4.3 Calibration and Validation

The statistics presented within this report are separated

Site Name	Site No.	Start Date	End date	Missing $(\%)$	Bad (%)	Total $(\%)$	Aggregation Start	Aggregation End	Missing vears	Sim Years
Adelaide	023034	13/01/1967	3/04/2002	1.2	3.7	4.9	1/01/1967	31/12/2001	Nil	35
Alice Springs	015590	23/06/1951	12/03/2002	3.5	2.3	5.8	1/01/1952	31/12/2001	Nil	50
Brisbane	040214	1/01/1908	24/06/1994	2.4	3.4	5.8	1/01/1908	31/12/1993	1909, 1910	84
Cairns	031011	10/09/1942	30/09/2002	1.3	6.0	7.3	1/01/1943	31/12/2001	Nil	59
Darwin	014015	16/09/1953	17/09/2002	4.8	3.7	8.5	1/01/1954	31/12/2001	Nil	48
Hobart	094029	30/04/1911	1/10/2000	0.7	7.8	8.5	1/01/1912	31/12/1999	Nil	88
Melbourne	086071	30/4/1873	1/05/2003	12.3	4.3	16.6	1/1/1877	31/12/2002	1895, 1915-1924	115
Perth	009034	3/01/1946	28/04/1992	1.1	5.6	6.7	1/01/1946	31/12/1991	Nil	46
Sydney	066062	3/01/1913	9/10/2002	4.6	3.6	8.2	1/01/1913	31/12/2001	1918, 1920, 1994	86
Townsville	032040	3/03/1953	9/10/2002	0.1	3.8	3.9	1/01/1954	31/12/2001	Nil	48

Table 1. Raw 6-min Pluviograph Data Details.

Table 2. Daily Data Details for Aggregation Statistics.

Site Name	Site No.	Start Date	End date	Aggregation Start	Aggregation End	Sim Years
Adelaide	023034	1/03/1955	30/06/2003	1/01/1956	31/12/2002	47
Alice Springs	015590	1/11/1941	30/06/2003	1/01/1942	31/12/2002	61
Brisbane	040214	1/1/1840	30/06/1994	1/1/1887	31/12/1993	107
Cairns	031011	1/09/1942	30/06/2003	1/01/1943	31/12/2002	60
Darwin	014015	1/01/1941	30/06/2003	1/01/1941	31/12/2002	62
Hobart	094029	1/1/1882	30/06/2003	1/1/1894	31/12/2002	109
Melbourne	086071	1/4/1855	30/06/2003	1/1/1856	31/12/2002	147
Perth	009034	1/1/1876	1/04/1992	1/1/1880	31/12/1991	112
Sydney	066062	1/7/1858	30/06/2003	1/1/1859	31/12/2002	144
Townsville	032040	1/11/1940	30/06/2003	1/01/1941	31/12/2002	62

into two parts:

- *Calibration statistics*: Statistics related closely to the calibrated variables within each model which are used to verify that the calibration procedure is working correctly.
- *Validation statistics*: Statistics related to the models capabilities and not used in the calibration process; especially statistics important to the end user, which validate use of the model.

As the objective of this study is to compare two models using different calibration procedures, the discussion will focus on validation statistics. Calibration plots will not be discussed in detail.

4.3.1 Calibration Statistics

Calibration plots for each model are provided with Appendices C and D for DRIP and NSRP respectively.

Appendix C can be downloaded at: www.catchment.crc.org.au/pdfs/technical200409_c.pdf

Appendix D can be downloaded at:

www.catchment.crc.org.au/pdfs/technical200409_d.pdf

These statistics (derived from the pluviograph record used in calibration) are presented on a site-by-site basis.

For DRIP, the raw event characteristics are presented in histogram form (interstorm duration, storm duration and intensity – e.g. Figures C.1(a) for Adelaide) with associated mean, standard deviation, skew, autocorrelation and correlation with other event characteristics (Figures C.1(b)-(c) for Adelaide). These were the events used in calibration and thus a close fit should be found to these statistics.

The NSRP calibration plots (Figure D.1 for Adelaide) show the observed and final fitted estimate of the statistics used in calibration (eg. mean rainfall, coefficient of variation, skewness, dry Proportion, transition probabilities and autocorrelation) at the various aggregation levels used in calibration. Again, it is expected that such statistics match the observed data closely in these plots.

4.3.2 Validation Statistics

Table 3 presents a list of the model validation statistics produced for both DRIP and the NSRP. The pluviograph record was used to derive sets of statistics at various aggregation levels, of which the 1, 6 and 24 hr statistics are presented. The daily record was used to derive statistics at daily, monthly and annual aggregation levels. 'Wet' statistics are included which represent that the statistics only included data bins with rainfall greater than 0.0 mm. Spell duration statistics are also presented (again with a 0.0 mm dry/wet threshold). A wetspell duration is defined here as consecutive rainfall bins above 0.0 mm. A dryspell is defined as consecutive (or singular) rainfall bins with 0.0mm rainfall. This differs from the interstorm duration defined in relation to DRIP events, as here there is no minimum interstorm duration involved.

Of the many statistics calculated, some important statistics were chosen to demonstrate the performance of each model. These results are presented in Appendices A and B for DRIP and the NSRP respectively. These important statistics comprised of several 'standard' rainfall model validation statistics, an extreme distribution plot and a plot of the annual rainfall distribution. All of the remaining statistics listed in Table 3 are presented within Appendices C and D on a site-by-site basis. These appendices can be downloaded from the CRC for Catchment Hydrology web site (see Section 4.3.1).

4.3.3 Standard Validation Statistics

The set of rainfall model validation statistics presented here comprised of 'standard' statistics in previous studies (Onof and Wheater, 1993, Cameron *et al.*, 2000) along with a few additional statistics (points 4, 8, 9, 10, 11). The following statistics were presented for the 1, 6 and 24 hr durations:

- 1. The dry probability.
- 2. The mean of the continuous rainfall series.
- 3. The standard deviation of the continuous rainfall series.
- 4. The coefficient of skew of the continuous rainfall series.
- 5. The lag 1 autocorrelation coefficient of the continuous rainfall series.
- 6. The mean of the dryspell duration.
- 7. The standard deviation of the dryspell duration.
- 8. The mean of the wetspell duration.
- 9. The standard deviation of the wetspell duration.
- 10.Dryspell-wetspell duration correlation coefficient.
- 11.Wetspell-dryspell duration correlation coefficient.

Table 3. Validation Statistics.

In this study the skew statistics were added for further validation, along with the various cross correlation statistics. The mean µ, standard deviation σ, coefficient of skewness κ, lag-1 autocorrelation ρ_x and cross correlation coefficient ρ_{xx} are estimated from the following equations;

$$
\hat{\mu}_x = \frac{1}{n} \sum_{i=1}^n x_i
$$
\n
$$
\hat{\sigma}_x = \sqrt{\frac{1}{(n-1)} \sum_{i=1}^n (x_i - \hat{\mu}_x)^2}
$$
\n
$$
\hat{\kappa}_x = \frac{n}{(n-1)(n-2)\hat{\sigma}_x^3} \sum_{i=1}^n (x_i - \hat{\mu}_x)^3
$$
\n
$$
\hat{\rho}_x = \frac{1}{(n-1)\hat{\sigma}_x^2} \sum_{i=1}^{n-1} (x_i - \hat{\mu}_x)(x_{i+1} - \hat{\mu}_x)
$$
\n
$$
\hat{\rho}_{xy} = \frac{1}{n\hat{\sigma}_x \hat{\sigma}_y} \sum_{i=1}^n (x_i - \hat{\mu}_x)(y_i - \hat{\mu}_y)
$$

In the above equations, x_t (and y_t) represents the variable of interest and *n* the number of data values.

4.3.4 Extreme Rainfall Statistics

Intensity-Frequency-Duration plots are produced to examine the ability of the models to reproduce the distribution of annual extremes. The IFD curves plot the maximum intensity of each year for a given duration.

4.3.5 Annual Rainfall Distribution

A plot of the annual rainfall amounts (versus exceedance probability) provides a test of the model ability to produce annual variability within the observed record.

5. Results

As there are many plots and sites produced as part of this study, not all plots will be discussed, but do provide the reader with further tools for stricter assessment of the models capabilities. Discussion of results here will focus on the model validation statistics presented in Appendix A and B. Attention will be given to occasions where a particular model does not reproduce a given statistic (there is an obvious bias across all sites). Also, if a model reproduces an observed statistic poorly for a single site, it will also be discussed. For convenience MX $(X=1,...,12)$ will be used to denote months of the year, where January corresponds to M1. Results are presented on a statistic-by-statistic basis, with Figure A.1 for DRIP being comparable to Figure B.1 for the NSRP. For figure numbers corresponding to each statistic see Table 3.

For all statistics plotted, the observed values are plotted as a point value – for the 1, 6 and 24 hr results the symbols \circ , Δ and + are used respectively. Simulated median (a thick line) and 90% confidence limits (thin lines) are also plotted. These simulation statistics were calculated by ranking the repeated simulation results. Results will be judged by discussing how many of the observations lie within the confidence limits. The more observations that lie within the confidence limits the better performing the model. If all observations lie on one side of the confidence limits, bias is present. Models will be judged on their performance over all sites, however some individual sites may provide exceptions.

5.1. 1, 6 and 24 Hr Standard Validation Statistics from Pluviograph Record

Dry probability: Seasonality of dry probability is reproduced well by DRIP (Figure A.1). The observed dry probability is however consistently above that simulated (often above the 5% exceedance quantile – eg. for every Melbourne result). This difference can be explained to some degree by the presence of missing or corrupted values within the record. As missing and corrupted values tend to be occasions where it is raining the overall dry probability estimate is biased upwards.

Seasonality of dry probability is not as well produced by the NSRP (Figure B.1 – e.g. 24 hr Brisbane and Sydney). Neither constant underestimation nor overestimation is observed across all sites. However, the simulated series does vary around that observed to a greater degree than for DRIP (e.g. Hobart). This variability is surprising considering the dry probability was used in calibration of the NSRP.

Mean: Mean rainfall is reproduced well for both models (Figures A.2 and B.2). The DRIP simulated median tends to be slightly above that observed. This is attributed to the observed estimate being biased downwards, due to the predominance of missing values being wet. The NSRP matches the median simulated mean very closely.

Standard deviation: The observations lie close to the simulated DRIP and NSRP median (and within the 90% confidence limits) for the majority of sites/months (Figures A.3 and B.3).

Skew: Historical skew values lie within the 90% confidence intervals for DRIP and NSRP for the majority of sites/months (Figures A.4 and B.4). The NSRP model tends to have tighter bounds on the skew (whilst still encompassing the observed values $-$ e.g. Brisbane and Perth), indicating that the skew is better identified by the NSRP model – presumably due to the NSRP explicitly using rainfall skew within calibration.

Autocorrelation: Although generally within the confidence limits, the DRIP simulations tend to show overestimation of 1 hr lag one autocorrelation (Figure A.5), and underestimation of 24 hr autocorrelation. NSRP reproduces the observed values well (Figure B.5), with the 1 hr simulation occasionally significantly overestimating that observed (e.g. Hobart). Again the NSRP bounds are tighter than that of DRIP, due to autocorrelation being explicitly used in calibration.

Dryspell duration mean: Dryspell mean is reproduced very well by DRIP for the 6 and 24 hr aggregation levels (Figure A.6). DRIP 1 hour dryspell median mean duration is consistently above that observed. This is possibly a result of dryspells terminating/starting with –ve values in the record being used in the observed calculation. This on average would shorten the observed mean dryspell length. The dryspell mean is reproduced very poorly

by the NSRP model (Figure B.6), especially at the 24 hr level (e.g. Brisbane, Cairns, Hobart, Melbourne, Sydney) – displaying constant overestimation.

Dryspell duration standard deviation: Dryspell standard deviation is reproduced well by DRIP (Figure A.7), being within the 90% confidence limits for the majority of observations. A slight overestimation is apparent at the 1 hr level, again attributed to the bias in the observed record. Marked overestimation of dryspell duration standard deviation occurs at all aggregation levels for the NSRP model (Figure B.7).

Wetspell duration mean: Seasonality of the wetspell mean is reproduced well by DRIP (Figure A.8). Slight overestimation occurs at both the 1 and 24 hr levels. NSRP tends to overestimate the 24 hr wetspell mean, and underestimate the 1 hr wetspell mean (Figure B.8).

Wetspell duration standard deviation: Wetspell standard deviation is reproduced reasonably by DRIP (Figure A.9). The NSRP wetspell standard deviation shows a greater degree of variability for the majority of sites (Figure B.9), whilst not matching the seasonality as well. Occasional months occur which show huge overestimation (eg. Brisbane M6, Hobart M6, M9).

Dryspell-wetspell duration correlation: It is noted that dryspell-wetspell duration correlations were generally reproduced well by DRIP (Figure A.10) – with correlations being generally small (0.1). NSRP tended to overestimate (Figure B.10 - e.g. Brisbane, Sydney).

Wetspell-dryspell duration correlation: Again DRIP reproduces the wetspell-dryspell correlations well (Figure A.11), while NSRP tended to underestimate (Figure B.11).

5.1.1 Discussion

Overall DRIP and the NSRP perform well for the mean, standard deviation and skew of rainfall. NSRP outperforms DRIP in terms of reproducing the autocorrelation. This is presumably a result of DRIP not having a mechanism for including autocorrelation from one event to the next, all autocorrelation is induced by within storm correlation, whereas NSRP uses autocorrelation within calibration. In terms of the wetspell and dryspell statistics presented, DRIP clearly outperforms the NSRP. DRIP consequently outperforms the NSRP in terms of dry probability also.

These results regarding the NSRP are somewhat surprising considering the dry probability was used in calibration. The spell results could be expected to have been better reproduced by the NSRP considering the transition probabilities were used in calibration too. It can only be concluded that the other statistics have a greater effect in calibration, in that it is easier to reproduce the other statistics. Although not presented in Appendix B, calibration plots of the NSRP show that of the calibration statistics used, the transition probabilities are the most poorly reproduced (especially the wet-to-wet transition probability). This problem could possibly be rectified by placing a greater weight within calibration on the transition probabilities if important in the application.

5.2 Intensity-Frequency-Duration Curves from Pluviograph Record

The annual maxima for a range of durations were extracted from the observed and simulated records. DRIP used a 6 minute moving bin width, while NSRP used a 1 hr bin width. This results in the annual maximum being greater on average for DRIP for a given duration, given the same observed record. This is not of consequence regarding the comparisons as consistent binning techniques for each model and data set were used – and it is relative differences between that observed and simulated that are important (rather than DRIP simulated vs. NSRP simulated).

5.2.1 DRIP

The IFD curves are presented for DRIP at the 0.1, 1, 6 and 24 hr levels (Figure A.12). For the 1, 6 and 24 hr aggregation levels the majority of sites observed values fall within the simulated bounds (even at high ARI's eg. 50 yrs). Some notable exceptions are the 1 hr Melbourne and Sydney simulations showing underestimation for ARI's in the range 2-10 yrs. DRIP's performance over the 1, 6 and 24 hr timescales is considered to be satisfactory. DRIP's 0.1 hr simulations gave varied results. For the tropical sites (Darwin, Cairns, Townsville and to a lesser extent Brisbane) the upper tail of the distributions is markedly overestimated.

5.2.2 NSRP

IFD curves are presented for the 1, 6 and 24 hr durations (Figure B.12). It is considered that the maxima distributions are preserved very well. IFD statistics at durations smaller than 1 hr are not presented as this was the minimum bin width for the model.

5.2.3 Discussion

NSRP outperforms DRIP in terms of IFD distributions. The 1, 6 and 24 hr IFD's were generally satisfactory for DRIP with the exception of Sydney and Melbourne which were underestimated for ARI's from 2-10 yrs. Although not presented for the NSRP, DRIP significantly overestimated IFD curves at the 0.1 hr level for some tropical sites. This may indicate that either too many short duration storms with high intensity are being generated, or alternatively the disaggregation scheme in DRIP is producing periods within storms that are too intense. As discussed in Section 2.3, the current exclusion of rainfall storm events with less than 1 hr of cumulative wet bins within disaggregation parameter estimation may be decreasing the quality of IFD reproduction. In some cases the DRIP simulated median at the 6 minute level is over twice that observed for the upper tail (eg. Brisbane, Cairns, Townsville). This could potentially result in overdesign if the model is used in such a case, if these short duration maximum intensities are important for a particular project.

5.3 Annual Rainfall Distribution from Daily Record

Annual rainfall distribution: DRIP reproduces annual rainfall distributions satisfactorily for such a model (Figure A.13), with the exception of Hobart and Perth. For the remaining sites the distributional shape is matched reasonably, however underestimation of annual variance is evident. The NSRP performs better than DRIP in terms of annual distributional shape (Figure B.13), with the majority of observations lying within the confidence limits. One exception to this quality of fit for the NSRP is for Perth, where the distribution is markedly underestimated. This could plausibly be due to the prevalence of missing values, possibly causing bias in the mean for particular months.

5.4 Standard Statistics from Daily Record

The statistics derived from the daily record provide a more accurate check of 24 hr statistics as the records have relatively few missing values. The dry probability and daily mean are especially important as the pluviograph record could possibly be biased by missing or corrupted values.

Daily dry probability: Converse to the results for 24 hrs using the pluviograph data, DRIP consistently overestimates daily dry probability (Figure A.14). The observed values typically lie on the 95% exceedance quantile. NSRP does not reproduce the seasonality of dry probability as well as DRIP (Figure B.14), whilst also showing general overestimation of daily dry probability.

Daily mean, standard deviation and skew: DRIP reproduces daily mean, standard deviation and skew of rainfall well (Figures A.15-17), with two exceptions. For Hobart the daily mean is generally overestimated, and for Perth the mean is underestimated. The standard deviation is markedly overestimated for Perth for M2. This result could again plausibly be attributed to the high prevalence of corrupted values within these records. The NSRP reproduces daily mean, standard deviation and skew of rainfall well for all sites (Figures B.15-17).

Daily autocorrelation: DRIP shows a bias towards underestimation of daily lag-one autocorrelation for the majority of sites (Figure A.18). The NSRP reproduces daily autocorrelation reasonably well (Figure B.18).

Daily dryspell mean and standard deviation: Dryspell means and standard deviations are marginally overestimated by DRIP (Figures A.19-20). The NSRP performs poorly, significantly overestimating for the majority of sites (Figures B.19-20).

Daily wetspell mean and standard deviation: Wetspell means and standard deviations are underestimated by DRIP (Figures A.21-22). The NSRP slightly overestimates the mean for the majority of sites, whilst also showing a large degree of variability not observed from month-to-month (Figures B.21-22).

5.4.1 Discussion

Overall DRIP and NSRP perform generally well for the mean, standard deviation and skew of daily rainfall. NSRP outperforms DRIP in terms of reproducing daily autocorrelation. DRIP and NSRP reproduce the observed daily dry probabilities reasonably well. However, DRIP shows bias in that it overestimates the dry probability overall. The dryspell statistics are produced comparatively well by DRIP, while also being marginally better for the wetspell statistics (the seasonality is reproduced reasonably – whilst not showing months that were greatly over- or underestimated).

6. Discussion

The study concentrated on evaluating two subdaily rainfall generation models with respect to:

- preserving various 'standard' 1, 6 and 24 hr rainfall model aggregation statistics using pluviograph data as input;
- preserving extreme rainfall statistics Intensity-Frequency-Duration curves; and
- preserving other statistics at greater timescales (daily, monthly and annual) using daily data as input.

DRIP and NSRP use different calibration techniques, which focus on reproduction of different variables from the observed record. Given that subdaily modelling is the focus of this paper, it is imperative that short timescale 'standard' statistics are reproduced well. Reproduction of statistics at greater timescales is less important, but desired. Finally, the IFD curves provide a good indication of the models ability of simulation of extremes.

The results indicate that both models adequately preserve the mean, standard deviation and skew of historical rainfall at 1, 6 and 24 hr time scales. The two models do differ in the quality of fit to the dry probability; DRIP follows the seasonality displayed in the historical record closely, whereas the NSRP model shows an inferior fit. DRIP shows a good fit to the mean and standard deviation of dryspell durations. NSRP again shows an inferior fit to these statistics. For wetspell mean durations the models are more evenly matched, however DRIP produces wetspell standard deviations more consistent with those observed than the NSRP. The NSRP does outperform DRIP in terms of serial autocorrelation of rainfall.

Of the daily statistics presented, both models perform well in reproducing daily mean, standard deviation and skew. NSRP outperforms DRIP in terms of reproducing daily autocorrelation. DRIP and NSRP reproduce the observed daily dry probabilities reasonably. However, DRIP shows bias in that it overestimates the dry probability overall.

NSRP tends to produce the annual rainfall distribution more satisfactorily than DRIP, with DRIP tending to underestimate the annual variance.

Importantly, NSRP reproduces the IFD curves very well for the timescales presented (1, 6 and 24 hr). DRIP allowed testing to a finer timescale $(0.1, 1, 6$ and 24 hrs). The majority of sites showed satisfactory reproduction of observed values over the 1, 6 and 24 hr range – with the notable underestimation of Sydney and Melbourne 1 hr maxima for ARI's from 2-10 yrs DRIP also severely overestimated the upper tail for the short durations (0.1 hrs) for some tropical sites.

7. Conclusion

Two stochastic rainfall models were applied to pluviograph data obtained from ten sites within Australia. These sites were chosen due to the relative length of records available, at least 45 years of data (with the exception of Adelaide). Other Bureau of Meteorology pluviograph sites (that have been digitised) throughout Australia have lengths typically in the range 15-25 years. Thus, this can be considered as being the best possible opportunity for identification of model parameters in Australian conditions.

The two models chosen for comparison were the Disaggregated Rectangular Intensity Pulse (DRIP) model of Heneker *et al.*, (2001), and the single site version of the Neyman-Scott Rectangular Pulse (NSRP) process model of Cowpertwait *et al.*, (2002). The models were evaluated on a monthly basis regarding their ability to reproduce certain 'standard' and extreme rainfall model statistics derived from the pluviograph record over a range of timescales (1, 6 and 24 hrs). Other daily, monthly and annual statistics derived from the longer daily rainfall series were also presented.

Of the shorter timescale standard statistics, DRIP performed more adequately. The NSRP model performed poorly for wetspell and dryspell statistics. DRIP's superior performance is attributed to DRIP being calibrated to interstorm durations and storm duration, which are closely related to mean wet and dryspell lengths. NSRP on the other hand is calibrated to a range of statistics, some of which may hinder the reproduction of these spell statistics.

Both models perform adequately at greater timescales of aggregation (for the statistics discussed). The NSRP performs better in regard to Intensity-Frequency-Duration curves. DRIP performing generally well with some exceptions for 1 hr maxima for ARI's from 2-10 yrs (Sydney, Melbourne), and overestimating the upper tail significantly at short timescales (0.1 hr) for some tropical sites.

Based on the comparison of the two current models, deficiencies in both models have been identified. The NSRP model reproduces many statistics well, however

performs poorly in regard to some wet and dry spell statistics. DRIP reproduces wet and dryspell characteristics well (as these are used in calibration), however performs poorly in terms of short duration IFD.

It is stressed that there are deficiencies in both approaches which could be addressed quite simply. Firstly greater weight could be placed on fitting the dry and transition probabilities for the NSRP model to force better reproduction of spell statistics. However, the somewhat difficult question of how much weight remains open. Likewise, DRIP is heavily parameterised currently with 30 parameters per month, compared to the NSRP's six. As mentioned in the description of DRIP, issues regarding intensity censuring and interstorm duration censuring (when terminated by a negative value) need to be addressed. Also, it is not known how the current DRIP intensity double fitting procedure would cope with less data (as would be the case for the majority of sites throughout Australia). The current exclusion of rainfall events with less than 1 hour of cumulative wet bins in the DRIP disaggregation calibration is expected to be a factor in the poor reproduction of short duration IFD distributions.

Further work is underway on both models by model authors to address these issues. It is not expected that one model will be able to be recommended over another, as both perform well for some statistics, whilst not for others. Rather, it is envisaged that a model is chosen for a particular study based on the statistics important to that study (eg. IFD vs. spell characteristics). Given the general adequacy of both models over the wide range of statistics presented it is however recommended that both models are adequate for use within the CRC for Catchment Hydrology's toolkit, with appropriate information on advantages and disadvantages.

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Appendix A: DRIP Validation Statistics

For all statistics plotted, the observed values are plotted as a point value – for the 1, 6 and 24 hr results the symbols \circ , Δ and + are used respectively. Simulated median (a thick line) and 90% confidence limits (thin lines) are also plotted (see Section 5).

A colour version of this Appendix is available at www.catchment.crc.org.au/pdfs/technical200409.pdf

APPENDIX A

Figure A.1 DRIP monthly dry probability: 1,6 & 24 hr statistics

Figure A.2 DRIP monthly mean: 1,6 & 24 hr statistics

Figure A.3 DRIP monthly standard deviation: 1,6 & 24 hr statistics

Figure A.4 DRIP monthly skew: 1,6 & 24 hr statistics

Figure A.5 DRIP monthly autocorrelation: 1,6 & 24 hr statistics

Figure A.6 DRIP monthly dryspell mean: 1,6 & 24 hr statistics

Figure A.7 DRIP monthly dryspell standard deviation: 1,6 & 24 hr statistics

Figure A.8 DRIP monthly wetspell mean: 1,6 & 24 hr statistics

Figure A.9 DRIP monthly wetspell standard deviation: 1,6 & 24 hr statistics

Figure A.10 DRIP monthly dryspell−wetspell correlation: 1,6 & 24 hr statistics

APPENDIX A

Figure A.11 DRIP monthly wetspell−dryspell correlation: 1,6 & 24 hr statistics

Annual exceedance probability (%) Annual exceedance probability (%) Figure A.12 DRIP 0.1,1.0,6.0 & 24.0hr Intensity−Frequency−Duration curves

APPENDIX A

Annual exceedance probability (%) Annual exceedance probability (%) Figure A.13 DRIP annual rainfall distribution from daily record

Figure A.14 DRIP monthly dry probability: daily statistics from daily record

Figure A.15 DRIP monthly mean: daily statistics from daily record

Figure A.16 DRIP monthly standard deviation: daily statistics from daily record

APPENDIX A

Figure A.17 DRIP monthly skew: daily statistics from daily record

Figure A.18 DRIP monthly autocorrelation: daily statistics from daily record

Figure A.19 DRIP monthly dryspell mean: daily statistics from daily record

Figure A.20 DRIP monthly dryspell standard deviation: daily statistics from daily record **47**

Figure A.21 DRIP monthly wetspell mean: daily statistics from daily record

Figure A.22 DRIP monthly wetspell standard deviation: daily statistics from daily record **49**

Appendix B: NSRP Validation Statistics

For all statistics plotted, the observed values are plotted as a point value – for the 1, 6 and 24 hr results the symbols \circ , Δ and + are used respectively. Simulated median (a thick line) and 90% confidence limits (thin lines) are also plotted (see Section 5).

A colour version of this Appendix is available at www.catchment.crc.org.au/pdfs/technical200409.pdf

APPENDIX B

Figure B.1 NSRP monthly dry probability: 1,6 & 24 hr statistics

Figure B.2 NSRP monthly mean: 1,6 & 24 hr statistics

APPENDIX B

Figure B.3 NSRP monthly standard deviation: 1,6 & 24 hr statistics

Figure B.4 NSRP monthly skew: 1,6 & 24 hr statistics

Figure B.5 NSRP monthly autocorrelation: 1,6 & 24 hr statistics

Figure B.6 NSRP monthly dryspell mean: 1,6 & 24 hr statistics

Figure B.7 NSRP monthly dryspell standard deviation: 1,6 & 24 hr statistics

Figure B.8 NSRP monthly wetspell mean: 1,6 & 24 hr statistics

APPENDIX B

Figure B.9 NSRP monthly wetspell standard deviation: 1,6 & 24 hr statistics

Figure B.10 NSRP monthly dryspell−wetspell correlation: 1,6 & 24 hr statistics

APPENDIX B

Figure B.11 NSRP monthly wetspell−dryspell correlation: 1,6 & 24 hr statistics

APPENDIX B

Annual exceedance probability (%) Annual exceedance probability (%) Figure B.12 NSRP 1.0,6.0 & 24.0hr Intensity−Frequency−Duration curves

APPENDIX B

Annual exceedance probability (%) Annual exceedance probability (%) Figure B.13 NSRP annual rainfall distribution from daily record

Figure B.14 NSRP monthly dry probability: daily statistics from daily record

Figure B.15 NSRP monthly mean: daily statistics from daily record

Figure B.16 NSRP monthly standard deviation: daily statistics from daily record

APPENDIX B

Figure B.17 NSRP monthly skew: daily statistics from daily record

Figure B.18 NSRP monthly autocorrelation: daily statistics from daily record

Figure B.19 NSRP monthly dryspell mean: daily statistics from daily record

Figure B.20 NSRP monthly dryspell standard deviation: daily statistics from daily record **71**

Figure B.21 NSRP monthly wetspell mean: daily statistics from daily record

Figure B.22 NSRP monthly wetspell standard deviation: daily statistics from daily record **73**

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