COOPERATIVE RESEARCH CENTRE FOR **CATCHMENT HYDROLOGY**

STOCHASTIC GENERATION OF DAILY RAINFALL AT A NUMBER OF SITES

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Report **05/7**
June 2005

Ratnasingham Srikanthan

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Preface

One of the goals of the Climate Variability Program in the Cooperative Research Centre (CRC) for Catchment Hydrology is to develop and test computer programs for generating stochastic climate data at time scales from less than one hour to one year and for point sites to large catchments. The appropriate models will be part of SCL (Stochastic Climate Library - a suite of stochastic climate data generation models), a product in the CRCs Modelling Toolkit (see www.toolkit.net.au/scl).

This report describes the development and testing of a multi-site daily rainfall model (multi-site two-part model nested in a monthly and annual model). The model can be used to generate stochastic daily rainfall data for many sites (or catchments) that preserve the statistical characteristics at each site as well as the rainfall correlations between sites.

The stochastic daily rainfall data can then be used to drive hydrological and system models to quantify the uncertainty in environmental systems associated with hydroclimatic variability. The two-part model is a model in SCL.

Francis Chiew
Program Leader - Climate Variability Program
CRC for Catchment Hydrology

Executive Summary

This report describes the generation of daily rainfall data at a number of sites. A multisite two-part model is nested in a monthly and annual model. A first order Markov chain is used to model the occurrence of rainfall and the spatial correlation in the occurrence process is handled by using correlated uniformly distributed random numbers. A two parameter Gamma distribution is used with correlated random numbers to obtain the rainfall depths. The generated daily rainfall at each site is input to a monthly model. The resulting monthly rainfall is input to an annual rainfall model. This process ensures that the monthly and annual characteristics are preserved. However, this did not improve the spatial correlations between monthly and annual rainfalls. The attempt to improve the spatial correlation of monthly and annual rainfall was not successful as the correlated noise terms violated the assumptions of the multi-site model formulation.

The model was applied to five catchments/regions with the number of rainfall sites varying from three to thirty. A comparison of the historical and generated statistics showed that the model preserves all the important characteristics of rainfall at the daily, monthly and annual time scales. Only the skewness of monthly rainfall and the spatial cross correlations at the monthly and annual time scales were not preserved well. The model is considered adequate as it preserves all the important daily parameters including the daily spatial cross correlations.

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1. Introduction

Daily rainfall is a major input to water resources and agricultural systems. As the historical record provides a single realisation of the underlying climate, stochastically generated data are used to assess the impact of climate variability on water resources and agricultural systems. Daily rainfall data generation at a single site is a well researched area in the hydrological and climatological literature (Buishand 1978; Chapman 1994, 1998, 2001; Harrold *et al*., 2003a,b; Rajagopalan *et al*., 1996; Sharma and Lall 1999; Srikanthan and McMahon 1985, 2001; Woolhiser 1992). However, for assessing hydrological and land management changes over larger regions, the spatial dependence between the weather inputs at different sites have to be accommodated. This is particularly important to the simulation of rainfall, which displays the largest variability in time and space. The model used to generate daily rainfall at a number of sites can be broadly grouped into four categories – conditional models, extension of Markov chain models, random cascade models and nonparametric models.

Conditional models generate the occurrence and the amount of rainfall using surface and upper air data (Zucchini and Guttorp, 1991; Bardossy and Plate, 1991, 1992; Wilson and Lettenmaier, 1993; Hughes *et al*., 1999; Charles *et al*., 1999). Wilks (1998) extended the familiar two part model, consisting of a two-state, first-order Markov chain for rainfall occurrences and a mixed exponential distribution for rainfall amounts, to generate rainfall simultaneously at multiple locations by driving a collection of individual models with serially independent but spatially correlated random numbers. He applied the model to 25 sites in the New York area. Jothityangkoon *et al*. (2000) constructed a space-time model to generate synthetic fields of space-time daily rainfall. The model has two components: a temporal model based on a first-order, four-state Markov chain which generates a daily time series of the regionally averaged rainfall and a spatial model based on nonhomogeneous random cascade process which disaggregates the regionally averaged rainfall to produce spatial patterns of daily rainfall. The cascade used to disaggregate the rainfall spatially is a product of stochastic and deterministic factors; the latter enables the model to capture systematic spatial gradients exhibited by measured data. Buishand and Brandsma (2001) used nearest neighbour resampling for multisite generation of daily precipitation and temperature at 25 stations in the German part of the Rhine Basin. Mehrotra and Sharma (2005) applied the k-nearest neighbour technique to simulate rainfall conditional upon atmospheric variables simultaneously at 30 stations around Sydney.

Conditional models are both data and computationally intensive. Besides, these were applied to one location and not tested adequately. The random cascade models also require a large amount of data to characterise the spatial dependence at different levels in the cascade as it generates rainfall data over a grid. The nonparametric model is being developed at the University of New South Wales by Mehrotra and Sharma (2005). The extended two part model of Wilks (1998) which is an extension of the Markov chain model appears to be a relatively simple model and at the same time, it has the potential to perform well. A comparison with two other approaches (hidden state Markov model and the k-nearest neighbour model) to model rainfall occurrence has shown that this approach performed the best (Mehrotra *et al*., 2005). Hence this method was chosen for testing and further enhancements.

In this report, the extended two-part model is nested in monthly and annual models and its performance is evaluated by applying it to five catchments/regions with the number of rainfall stations varying from three to thirty.

2. Rainfall Data

Daily rainfall data from five catchments/regions were used to develop and test the multi-site daily rainfall model. The number of stations varies from three to thirty. The number of rainfall stations in each catchment/region, their locations and details are given in the following sections.

2.1 Upper Woody Yaloak River Catchment

The Woady Yaloak River Catchment is located in southwest Victoria. The area of the catchment is 1,157 km^2. There are three rainfall stations near the catchment, but none is within the catchment (Figure 1). Eighty three years of rainfall data were used covering the period 1919 to 2001. Table 1 gives the details of the three rainfall stations used.

Table 1. Details of the Rainfall Stations Used in the Upper Woady Yaloak River Catchment.

No.	Name	Lat.	Long.	Mean Annual Rainfall (mm)	C_V of Annual Rainfall	Wet Days per Year
089002	Ballarat Aerodrome	-37.51	143.79	702	0.42	169
089024	Rokewood	-37.90	143.70	569	0.41	134
089025	Skipton	-37.68	143.36	623	0.36	138

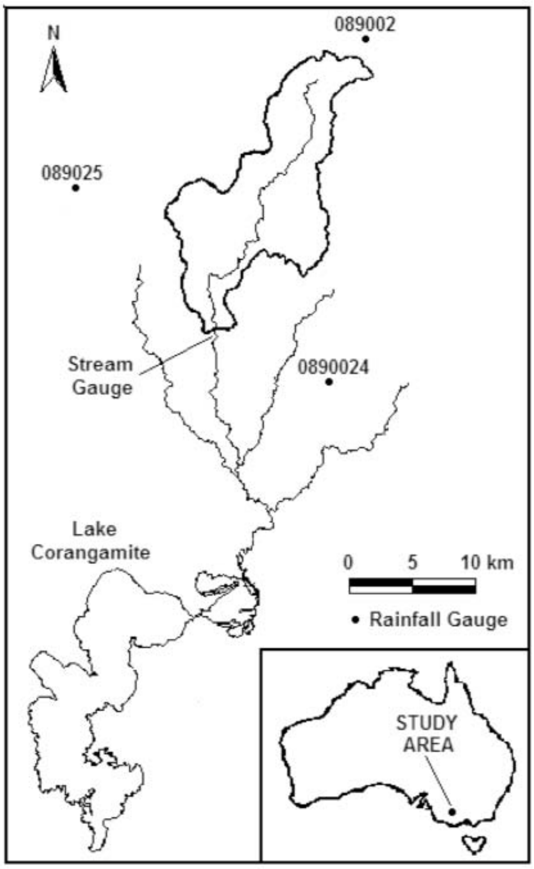

Figure 1. Location of Stations in the Upper Woady Yaloak River Catchment.

2.2 Yarra River Catchment

The Yarra River Catchment is located close to Melbourne and is one of the water supply catchments for Melbourne Water. There are ten rainfall stations located within the catchment which has an area of 3,957 km^2 (Figure 2). Forty one years of rainfall data were used covering the period 1955 to 1995. The mean annual rainfall varies from 635 mm to 1437 mm while the number of wet days varies from 90 to 252 (Table 2).

Table 2. Details of the Rainfall Stations Used in the Yarra River Catchment.

No.	Name	Lat.	Long.	Mean Annual Rainfall (mm)	C_V of Annual Rainfall	Wet Days Per Year
86027	Croydon	-37.79	145.28	922	0.15	153
86070	Maroondah Weir	-37.65	145.55	1136	0.16	179
86071	Melbourne R.O.	-37.81	144.97	662	0.19	148
86073	Mickleham	-37.56	144.88	635	0.21	90
86074	Mitcham	-37.83	145.19	871	0.17	178
86090	O'Shannassy	-37.71	145.79	1437	0.16	189
86096	Preston Reservoir	-37.73	145.01	716	0.18	159
86106	Silvan	-37.83	145.43	1256	0.15	252
86117	Toorourrong Reservoir	-37.48	145.15	829	0.19	158
86125	Whittlesea	-37.50	145.12	719	0.20	125

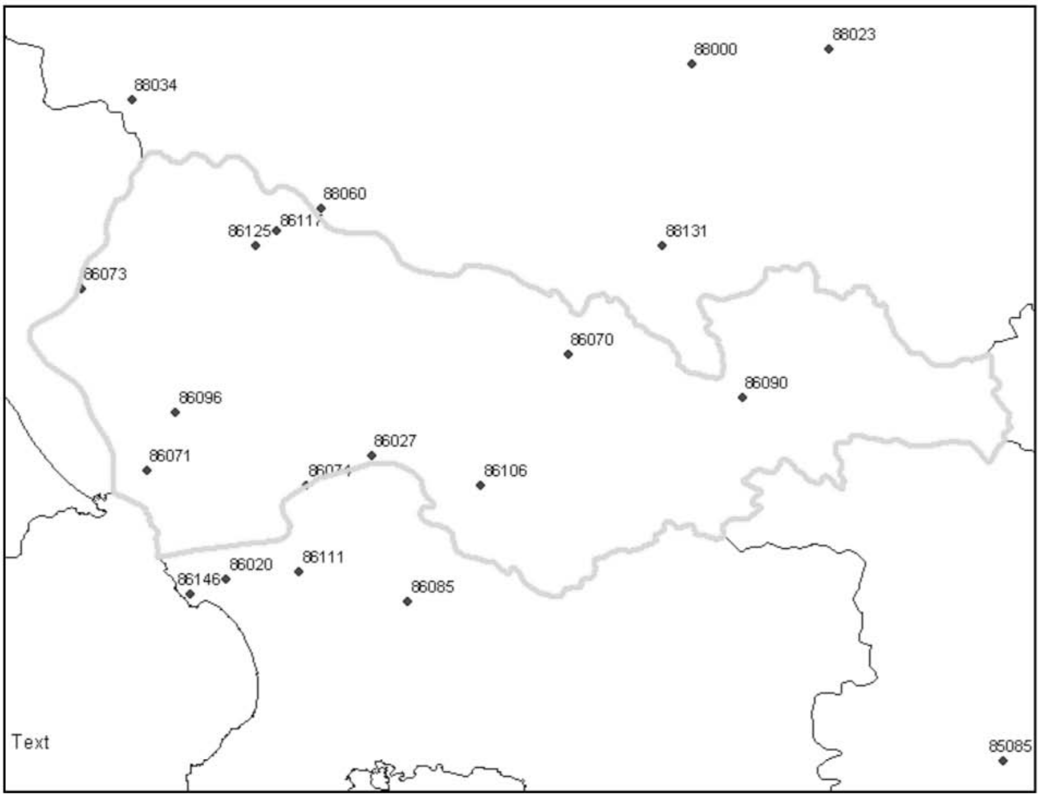

Figure 2. Location of Stations in the Yarra River Catchment.

2.3 Murrumbidgee River Catchment

The Murrumbidgee River Catchment is located in southern New South Wales and the Australian Capital Territory lies within the catchment. The area of the catchment is 81,563 km^2. Thirty stations are selected and their details and locations are given in Table 3 and Figure 3 respectively. One hundred and ten years of rainfall data were used covering the period 1890 to 1999. The mean annual rainfall varies from about 340 mm to 970 mm while the average number of wet days per year varies from 57 to 109.

Table 3. Details of the Rainfall Stations Used in the Yarra River Catchment.

No.	Name	Lat.	Long.	Mean Annual Rainfall (mm)	C_V of Annual Rainfall	Wet Days Per Year
49048	Balranald (Tillara)	-34.64	143.05	340	0.34	57
70014	Canberra AMO	-35.30	149.20	640	0.27	95
70028	Yass (Derringullen)	-34.74	148.89	756	0.27	96
70032	Fairlight Station	-35.23	148.91	879	0.28	95
70054	Cooma (Kiaora)	-36.20	149.06	561	0.28	90
70064	Michelago (Soglio)	-35.68	149.16	666	0.28	79
71021	Jindabyne (Lynwood)	-36.49	148.58	615	0.24	78
72008	Tarcutta (Wollumbi)	-35.39	147.57	669	0.27	82
72013	Carabost Forest HQ	-35.65	147.80	967	0.25	91
72044	Tumut 1 (Capper Street)	-35.32	148.23	843	0.25	109
72049	Woomargama (Estate)	-35.86	147.29	792	0.27	81
72150	Wagga AMO	-35.16	147.46	592	0.27	94
73007	Burrinjuck (Dam)	-35.00	148.60	988	0.29	106
73015	Gundagai	-35.07	148.10	763	0.28	100
73041	Wombat (Tumbleton)	-34.41	148.18	740	0.28	87
73124	Eurongilly (Bundaleer)	-34.93	147.77	574	0.29	70
74017	Tootal (Bryntirion)	-35.29	146.97	546	0.29	80
74062	Leeton Caravan Park	-34.57	146.41	451	0.31	74
74087	Urana (Nowranie)	-35.33	146.03	415	0.31	54
74108	Darlington Point (Tubbo)	-34.63	146.09	401	0.36	59
74115	Walbundrie PO	-35.68	146.73	556	0.29	78
74128	Deniliquin PO	-35.53	144.95	415	0.31	82
75006	Binya PO	-34.23	146.34	437	0.32	62
75020	Mallan (Niemur Valley)	-35.16	143.87	349	0.33	64
75028	Griffith CSIRO	-34.32	146.07	417	0.31	75
75029	Carrathool (Gum Creek)	-34.63	145.35	391	0.34	62
75031	Hay PO	-34.52	144.85	393	0.34	69
75049	Maude (Nap Nap)	-34.45	144.17	342	0.36	54
75050	Naradhan (Marshall)	-33.61	146.32	470	0.32	59
75056	Booroorban (Ramsay)	-34.94	144.74	373	0.36	51

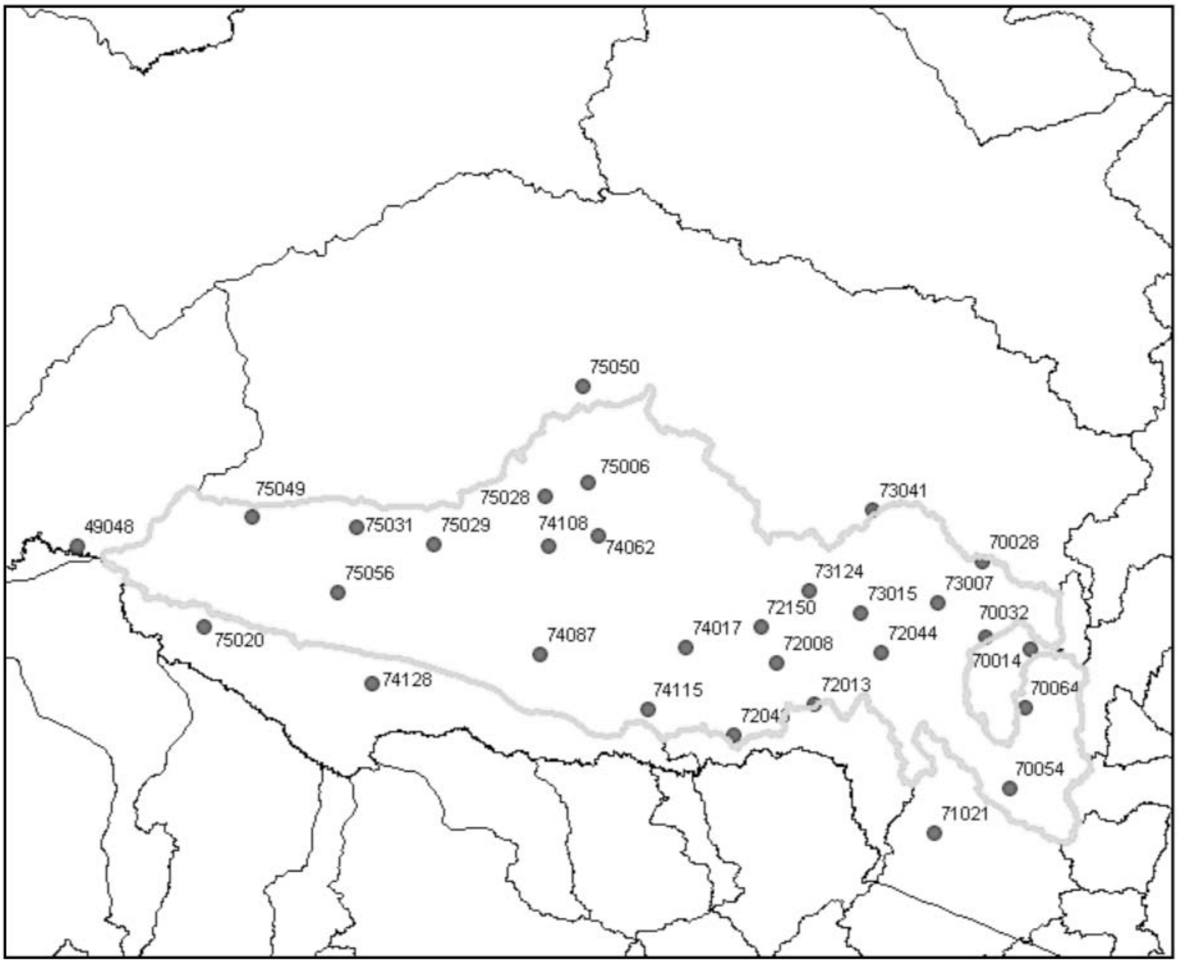

Figure 3. Location of Stations in the Upper Murrumbidgee River Catchment.

2.4 Goulburn-Broken River Catchment

The Goulburn-Broken River Catchment is located in northern Victoria and has an area of 23,595 km^2. Twenty eight rainfall stations are within or nearby the catchment and their locations are shown in Figure 4. Thirty five years of rainfall data were used covering the period 1961 to 1995. Unlike the other areas included in this study, there was a need for sub-catchment rainfall for a study at The University of Melbourne. The catchment was sub-divided into four sub-catchments, namely, Broken, Eildon, Goulburn and Trawool. The areal average rainfall for the sub-catchments was obtained by simple arithmetic mean. Daily rainfall data were generated for the four sub-catchments and the statistics were compared with the corresponding statistics obtained from the historical sub-catchment rainfall. The characteristics of annual rainfall for the sub-catchments are presented in Table 4.

Table 4. Characteristics of Annual Rainfall for the Sub-catchments.

Sub-catchment	Mean Annual Rainfall (mm)	C_V of Annual Rainfall	Wet Days Per Year
Broken	804	0.29	163
Eildon	901	0.19	230
Goulburn	642	0.24	197
Trawool	994	0.20	220

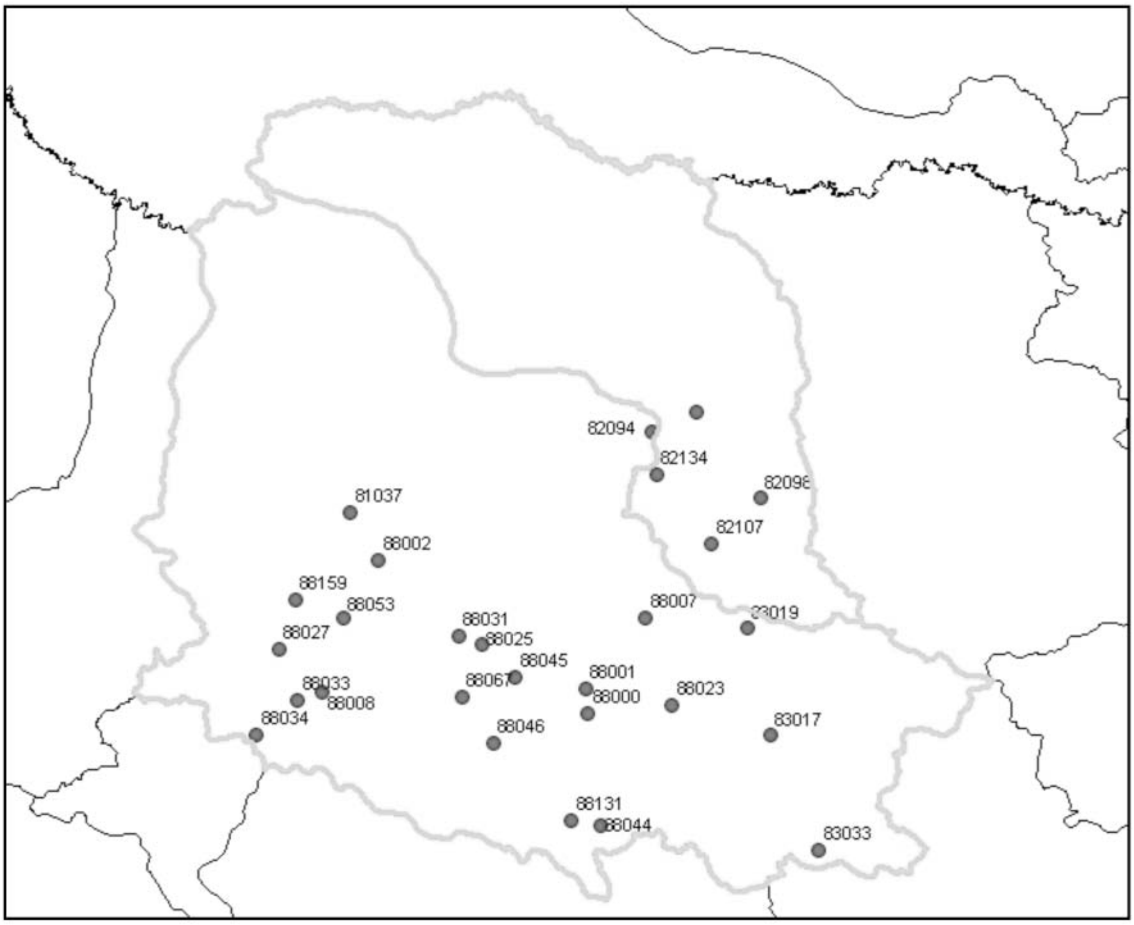

Figure 4. Location of Stations in the Upper Goulburn-Broken River Catchment.

2.5 Sydney Region

The Sydney region extends from Newcastle in the north to Canberra in the south. This region was selected primarily to compare results with another similar study carried out at the University of New South Wales (Mehrotra and Sharma 2005). Thirty rainfall stations are in the region. The location of the rainfall stations are shown in Figure 5. Forty three years of rainfall data were used covering the period 1960 to 2002. Table 5 gives the details of the rainfall stations. The mean annual rainfall varies from about 600 mm to 1300 mm and the average annual number of wet days varies from 70 to 160.

Table 5. Details of the Rainfall Stations used in the Sydney Region.

No.	Name	Lat.	Long.	Mean Annual Rainfall (mm)	C_V of Annual Rainfall	Wet Days Per Year
1	Pokolbin JH	-32.83	151.30	659	0.25	111
2	Glaucester	-31.72	151.80	650	0.25	97
3	Branxton	-32.63	151.42	826	0.23	94
4	Elong	-32.12	149.03	624	0.25	110
5	Jerrys Plain	-32.50	150.90	619	0.25	70
6	Merriwa Tunbridge	-32.22	150.23	924	0.27	93
7	Wyanga Composite	-32.47	148.15	677	0.26	88
8	Mittagong Kia Ora	-34.47	150.50	1294	0.36	145
9	Orange Rp	-33.38	149.12	659	0.20	102
10	Rlystone Kel	-32.87	150.30	1425	0.26	161
11	Wyong	-33.30	151.42	1036	0.28	122
12	Wyanga Dam	-33.97	148.95	611	0.21	70
13	Nerriga Composite	-35.12	150.08	924	0.32	95
14	Richmond	-33.62	150.75	710	0.28	91
15	Sydney R. O.	-33.87	151.20	878	0.22	102
16	Wellington Res Stn	-32.50	148.97	794	0.32	121
17	Parkes Mac Street	-33.15	148.17	901	0.25	139
18	Mudgee	-32.60	149.60	641	0.26	95
19	Williamtown AMO	-32.80	151.83	604	0.28	75
20	Murrurundi PO	-31.77	150.83	780	0.24	111
21	Canberra	-35.32	149.20	861	0.27	131
22	Jervis Bay	-35.10	150.80	804	0.23	100
23	Lucas Heights	-34.05	150.98	1261	0.27	138
24	Katoomba	-33.72	150.30	1129	0.25	142
25	Bathurst Agri Stn	-33.43	149.57	526	0.30	58
26	Gulgong	-32.37	149.53	705	0.26	108
27	Peak Hill	-32.72	148.18	1198	0.24	121
28	Yass Composite	-34.83	148.92	681	0.25	81
29	Goulburn Pomeroy	-34.65	149.50	607	0.26	87
30	Boorowa PO	-34.43	148.72	690	0.24	107

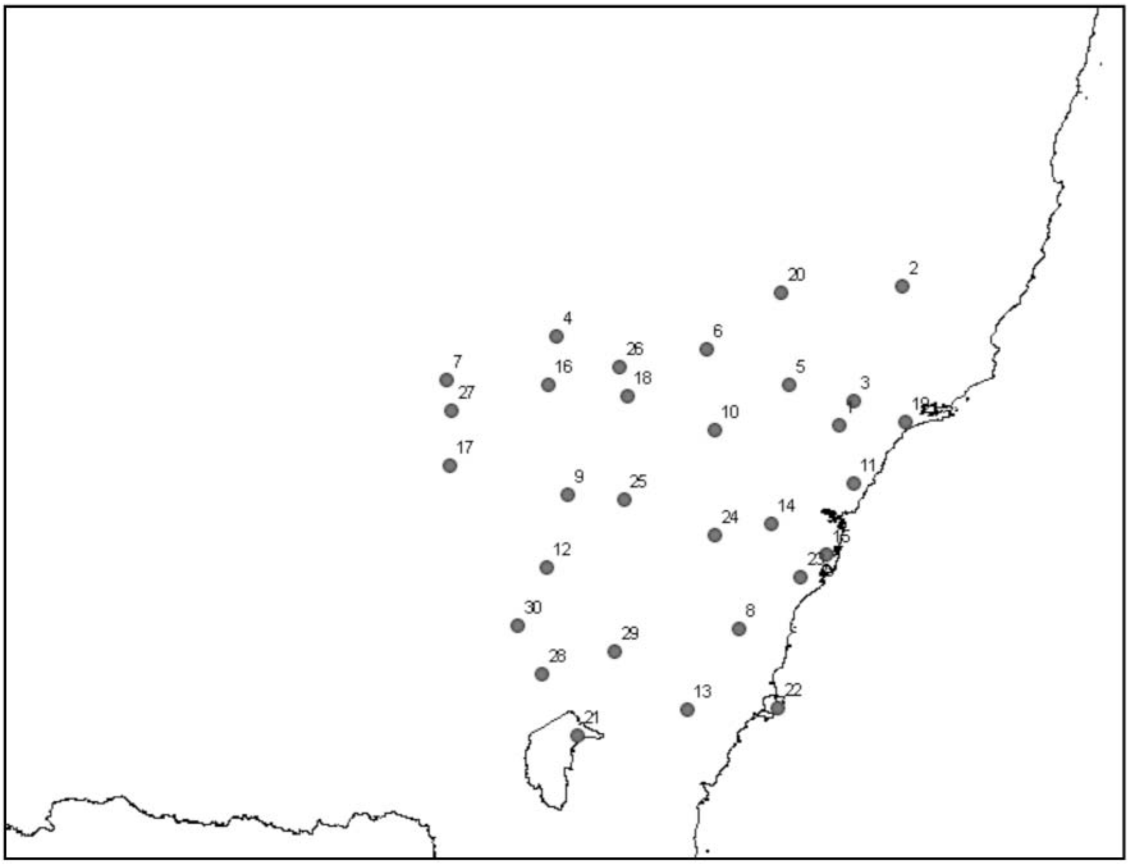

Figure 5. Location of Stations in the Sydney Region.

3. Multisite Two-part Model

Wilks (1998) extended the familiar two part model, consisting of a two-state, first-order Markov chain for rainfall occurrences and a mixed exponential distribution for rainfall amounts, to generate rainfall simultaneously at multiple locations by driving a collection of individual models with serially independent but spatially correlated random numbers. Individual models are fitted to each of the N sites first. The collection of individual site models are driven with vectors of uniform [0,1] variates $\mathbf{u}_t$ and $\mathbf{v}_t$ whose elements, $u_t(k)$ and $v_t(k)$ respectively, are correlated so that $corr[u_t(k), u_t(l)] \neq 0$ and $corr[v_t(k), v_t(l)] \neq 0$, and are serially and mutually independent $corr[u_t(k), v_t(l)] = corr[u_t(k), u_{t+1}(l)] = corr[v_t(k), v_{t+1}(l)] = 0$. Non-zero correlations among the elements of $\mathbf{u}_t$ and $\mathbf{v}_t$ result in inter-site correlations between the generated rainfall sequences.

3.1 Rainfall Occurrence Model

A first-order two-state Markov chain is used to determine the occurrence of rainfall. For each site *k*, the Markov chain has the two transition probabilities: $p^k_{W|D}$, the conditional probability of a wet day given that the previous day was dry; $p^k_{W|W}$, the conditional probability of a wet day given that the previous day was wet. The unconditional probability of a wet day for the site *k*, can be derived as:

$$\pi^k = \frac{p^k_{W|D}}{1 + p^k_{W|D} - p^k_{W|W}} \qquad (1)$$

Given a network of *N* locations, there are *N(N - 1)/2* pair wise correlations that should be maintained in the generated rainfall occurrences. This is achieved by using correlated uniform random numbers (u_t) in simulating the occurrence process. The uniform variates $u_t(k)$ can be derived from standard Gaussian variates $w_t(k)$ through the transformation:

$$u_t(k) = \Phi([w_t(k)] \qquad (2)$$

where:

$\Phi[.]$ indicates the standard normal cumulative distribution function. Let the correlation between the Gaussian variates, w_t, for the station pair *k* and *1* be:

$$\omega(k,1) = Corr[w_t(k), w_t(1)] \qquad (3)$$

Together with the transition probabilities for stations *k* and *1*, a particular $\omega(k,l)$ will yield a corresponding correlation between the synthetic binary series $(\boldsymbol{X}_t)$ for the two sites.

$$\xi(k,1) = Corr[X_t(k), X_t(1)] \qquad (4)$$

Let $\xi^0(k,1)$ denote the observed value of $\xi(k,1)$, which will have been estimated from the observed binary series $X^0_t(k)$ and $X^0_t(l)$ at stations *k* and *1*. Hence the problem reduces to finding the *N(N-1)/2* correlations of $\omega(k,l)$ which together with the corresponding pairs of transition probabilities reproduces $\xi^0(k,1) = \xi(k,1)$ for each pair of stations. Direct computation of $\omega(k,l)$ from $\xi^0(k,1)$ is not possible. In practice, one can invert the relationship between $\omega(k,l)$ and $\xi(k,1)$ using a nonlinear root finding algorithm or obtain $\omega(k,l)$ by simulation. In this report, the correlation between the corresponding normal variates is obtained by an iterative method using simulation and the method of bisection.

Realisations of the vector $\boldsymbol{w_t}$ may be generated from the multivariate normal distribution having mean vector $\mathbf{0}$ and variance-covariance matrix $\mathbf{\Omega}$, whose elements are the correlations $\omega(k,l)$.

The multivariate normal variates are generated from:

$$\boldsymbol{w_t} = \boldsymbol{B}\boldsymbol{\varepsilon}_t \qquad (5)$$

where:

$\boldsymbol{B}$ is a coefficient matrix and $\boldsymbol{\varepsilon}_t$ independent normal vector.

The coefficient matrix is obtained from:

$$\boldsymbol{B}\boldsymbol{B}^T = \mathbf{\Omega} \qquad (6)$$

The elements of $\boldsymbol{B}$ can be obtained by Cholesky's decomposition for a small number of rainfall stations (up to 5). For a larger number of rainfall stations, the Cholesky's decomposition fails and the elements of ***B*** can be obtained by singular value decomposition. The Cholesky's decomposition will result in an exact ***B*** matrix where as the singular value decomposition will result in an approximate ***B*** matrix. The seasonality in daily rainfall occurrence is taken into account by considering each month separately.

3.2 Rainfall Amount Model

The rainfall amounts on wet days are generated by using a Gamma distribution (Srikanthan, 2004) whose probability density function for site *k* is given by:

$$f(x^k) = \frac{(x^k / \beta^k)^{\alpha^k - 1} \exp(-x^k / \beta^k)}{\beta^k \Gamma(\alpha^k)} \qquad (7)$$

where:

α^k is the shape parameter and β^k the scale parameter. The mean and variance of the Gamma distribution are given by:

$$\mu(x^k) = \alpha^k \beta^k$$
$$\sigma^2(x^k) = \alpha^k \beta^{k2} \qquad (8)$$

The spatial correlation in the daily rainfall amounts is preserved by using a vector of correlated uniform variates $\boldsymbol{v}_t$. As in the rainfall occurrence model, it is convenient to obtain the elements of this vector from a corresponding realisation of correlated standard normal variates $z_t(k)$ as $v_t(k) = \Phi[z_t(k)]$. This vector $\boldsymbol{z}_t$ is drawn from a multivariate normal distribution with mean 0 and variance-covariance matrix ***Z***, whose elements are:

$$\xi(k,1) = Corr[z_t(k), z_t(1)] \qquad (9)$$

As was the case of **Ω**, direct computation of ***Z*** is not feasible since the z_t are not observed. The correlations in Equation (9) can be estimated using a similar procedure to the one used in the rainfall occurrence model. However, the response surface for the rainfall amount is flat and it may not be possible to obtain the correlation necessary between the normal variates (Wilks 1998). Under these circumstances, the correlation between the normal variates is obtained by multiplying the correlation between the rainfall amounts by 1.05. This adjustment seems to work well for the present study.

The correlated multivariate normal variates are obtained from independent normal variates as above.

The generated daily rainfall amounts when aggregated into monthly and annual totals will not in general preserve the monthly and annual characteristics. Hence, the daily amount model is nested in a monthly and annual model. This will only improve the monthly and daily characteristics of the generated rainfall and will have no effect on the spatial correlation for the monthly and annual rainfall.

Once the daily rainfall is generated for a month, the monthly rainfall is obtained by summing the daily rainfall values. For each site, the generated monthly rainfall value, $\tilde{X}_i^k$, is modified by using a monthly model (Srikanthan 2004) to preserve the monthly characteristics.

$$\frac{X_i^k - \mu(X_i^k)}{\sigma(X_i^k)} = \rho_{i,i-1}^k \frac{X_{i-1}^k - \mu(X_{i-1}^k)}{\sigma(X_{i-1}^k)} + (1 - \rho_{i,i-1}^k{}^2)^{1/2} \frac{\tilde{X}_i^k - \mu'(X_i^k)}{\sigma'(X_i^k)} \qquad (10)$$

where:

$\rho^k_{i,i-1}$ is the correlation coefficient between months *i* and *i-1*. The theoretical mean and variance of the rainfall total, X_i^k, over a month of *N* days is given by Katz (1985) as:

$$\mu'(X^k) = N\pi^k \alpha^k \beta^k \qquad (11)$$

$$\sigma'^2(X^k) = N\pi\alpha^k\beta^{k^2}\left[1+\alpha^k(1-\pi^k)\frac{1+p^k_{W|W}-p^k_{W|D}}{1-p^k_{W|W}+p^k_{W|D}}\right] \quad (12)$$

The subscript i for all the variables in Equations (11) and (12) is omitted for clarity. The generated daily rainfall data is multiplied by the ratio $X^k_i / \tilde{X}^k_i$. Once the values for the twelve months of a year *(j)* have been generated, the generated monthly values can be aggregated to obtain the annual value. The aggregated annual value, $\tilde{Z}^k_j$, is modified by using a lag one autoregressive model to preserve the annual characteristics.

$$\frac{Z^k_j - \mu(Z^k)}{\sigma(Z^k)} = \rho(Z^k)\frac{Z^k_{j-1} - \mu(Z^k)}{\sigma(Z^k)} + [1-\rho^2(Z^k)]^{1/2}\frac{\tilde{Z}^k_j - \mu'(Z^k)}{\sigma'(Z^k)} \quad (13)$$

where:

ρ(Z^k) is the lag one autocorrelation coefficient at site *k*. If the annual rainfall data exhibits significant skewness, then the noise term in Equation (13) is modified by using the Wilson-Hilferty transformation (1931). The theoretical values of the mean and variance of annual rainfall (Appendix A) obtained by the aggregated monthly rainfall are given by:

$$\mu(Z^k) = \sum_{j=1}^{12}\mu(X^k_j) \quad (14)$$

$$\begin{aligned}\sigma^2(Z^k) \approx {} & \sum_{j=1}^{12}\sigma^2(X^k_j) + 2\sum_{j=2}^{12}\sigma(X^k_j)\sigma(X^k_{j-1})\rho^k_{j,j-1} \\ & + 2\sum_{j=3}^{12}\sigma(X^k_j)\sigma(X^k_{j-2})\rho^k_{j,j-1}\rho^k_{j-1,j-2} \\ & + 2\sum_{j=4}^{12}\sigma(X^k_j)\sigma(X^k_{j-3})\rho^k_{j,j-1}\rho^k_{j-1,j-2}\rho^k_{j-2,j-3}\end{aligned} \quad (15)$$

The generated monthly rainfall value is multiplied by the ratio $Z^k_i / \tilde{Z}^k_i$. This will preserve the annual characteristics. The modified monthly rainfall values are used to adjust the daily rainfall values. Rather than adjusting the daily rainfall values twice, the adjustment to the daily rainfall values can be carried out in one step by multiplying the generated rainfall values for each month *(i)* by the ratio $X^k_i Z^k_i / \tilde{X}^k_j \tilde{Z}^k_j$.

An attempt was made to nest the daily amount model into multi-site monthly and annual models, but the results were poor. The reason for this is that the noise terms obtained from the generated rainfall are correlated and this inflates the variance of the monthly and annual rainfall. The multi-site model formulation assumes that the noise terms are independent and hence the poor results.

4. Model Evaluation

The model is evaluated by using a number of statistics at the daily, monthly and annual levels.

The following parameters are used to evaluate the generated annual rainfall data at each site:

- Mean ($\overline{x}$)
- Standard deviation (s)
- Coefficient of skewness (g)
- Lag one autocorrelation coefficient (r)
- Maximum
- Minimum
- Adjusted range
- Low rainfall sums of 2, 5, 7 and 10-year duration
- Annual number of wet days

In addition, cross correlation between the annual rainfall data is also compared.

The above parameters are estimated from a number of replicates each of length equal to the historical record. The first four items for each site *i* are estimated from the following expressions.

$$\overline{x}^i = \frac{1}{n}\sum_{t=1}^{n} x_t^i \tag{16}$$

$$s^i = \sqrt{\frac{1}{(n-1)}\sum_{t=1}^{n}(x_t^i - \overline{x}^i)^2} \tag{17}$$

$$g^i = \frac{n}{(n-1)(n-2){s^i}^3}\sum_{t=1}^{n}(x_t^i - \overline{x}^i)^3 \tag{18}$$

$$r^i = \frac{1}{(n-1){s^i}^2}\sum_{t=1}^{n-1}(x_{t+1}^i - \overline{x}^i)(x_t^i - \overline{x}^i) \tag{19}$$

$$R^i = \max\{D_k\} - \min\{D_k\} \tag{20}$$

where:

$$D_k = \sum_{t=1}^{k}(x_t^i - \overline{x}^i)$$

The cross correlation between the rainfall at sites *i* and *j* are obtained from:

$$r^{ij} = \frac{1}{(n-1)s^i s^j}\sum(x_t^i - \overline{x}^i)(x_t^j - \overline{x}^j) \tag{21}$$

The following monthly parameters are used to evaluate the generated monthly climate data at each site:

- Mean
- Standard deviation
- Coefficient of skewness
- Correlation coefficient between successive months
- Maximum
- Minimum

In addition, cross correlation between the monthly climate variables is also compared.

The following daily parameters are used to evaluate the generated daily rainfall data at each site:

- Mean number of wet days
- Mean
- Standard deviation
- Coefficient of skewness
- Correlation between rainfall amount and wet spell length
- Mean, standard deviation and skewness of dry spell length
- Mean, standard deviation and skewness of wet spell length
- Maximum dry and wet spell lengths

In addition, cross correlation between the daily rainfall depths and the log odds ratio for the rainfall occurrence are compared. The log-odds ratio for two sites is defined as:

$$lor = \log\left(\frac{p(D,D)\,p(W,W)}{p(D,W)\,p(W,D)}\right) \tag{22}$$

where:

p(D,D) - probability of both sites dry
p(W,W) - probability of both sites wet
p(D,W) - probability of first site dry and second site wet

5. Discussion of Results

One hundred replicates, each of length equal to the length of historical data were generated. The above parameters were estimated from each replicate and from these values the 2.5, 25, 50, 75, 97.5 percentile values and the mean were calculated.

5.1 Upper Woady Yaloak River Catchment

The log-odds ratio shown in Figure 6 shows that the spatial correlation between the rainfall occurrence processes at the three sites is preserved well. Also, the wet fraction is preserved well (Figure 7).

The historical and generated annual parameters for the Woady Yaloak catchment are given in Table 6 and are plotted in Figures B1 - B4 in Appendix B. This shows that the model preserves all the annual parameters except the minimum well. Both the historical and generated skewness are small and not significantly different from zero. The minimum value is slightly overestimated. However, all the low rainfall sums are satisfactorily preserved.

The historical and generated monthly parameters are shown in Figure B2 in Appendix B. The figure shows that the monthly parameters are satisfactorily preserved. A comparison of historical and generated daily rainfall parameters are shown in Figure B3. The mean number of wet days per month, maximum daily rainfall, mean and standard deviation of daily rainfall, dry and wet spell lengths are satisfactorily preserved. The skewness of daily rainfall and the correlation between rainfall depth and duration of wet spell are slightly underestimated while those of dry and wet spell lengths are not preserved well. Mean rainfall on wet days bounded on one side by a wet day is preserved while the mean on solitary wet days and wet days bounded by wet days on both sides are not preserved well.

The cross correlations between the rainfall occurrences at different sites are preserved well (Figure B4). The cross correlations between daily rainfall amounts are preserved well while those between monthly and annual rainfall are underestimated.

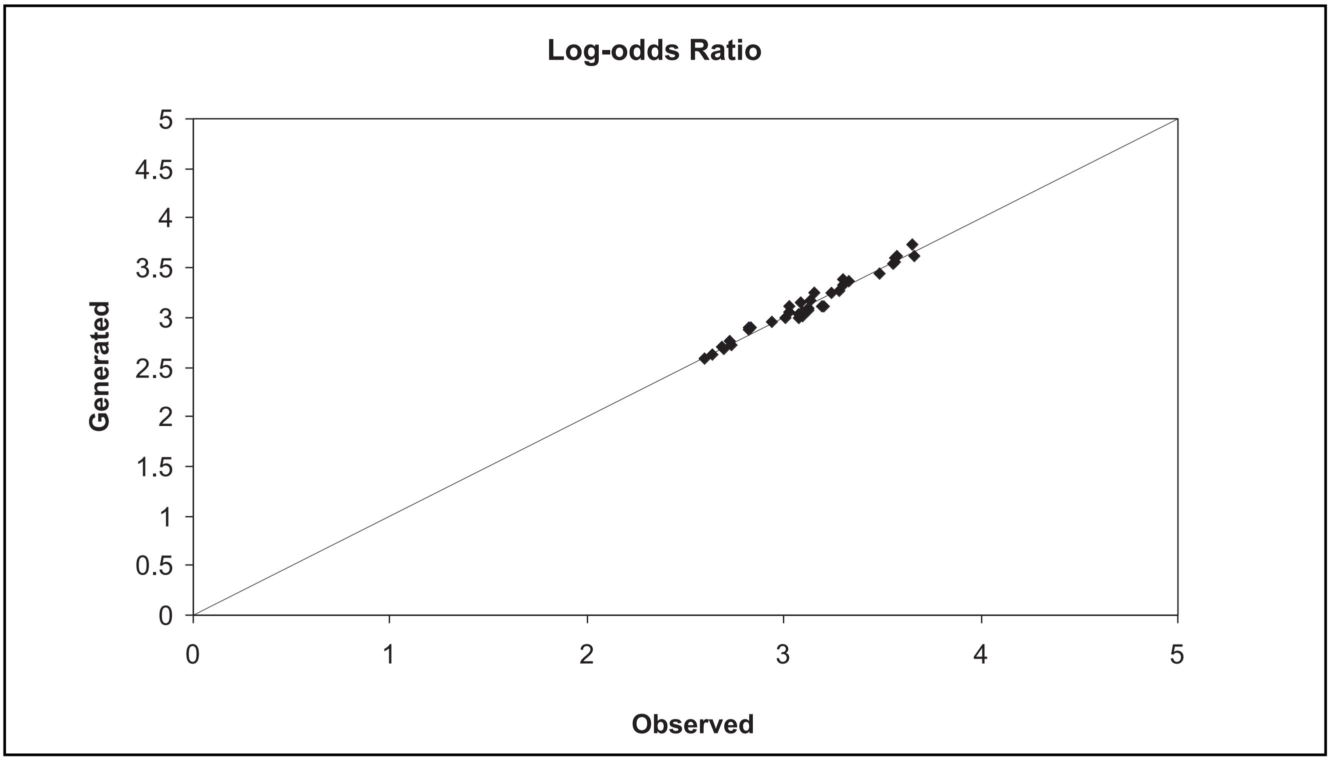

Figure 6. Log-odds Ratio for the Woady Yaloak Catchment (3 inter-site x 12 months).

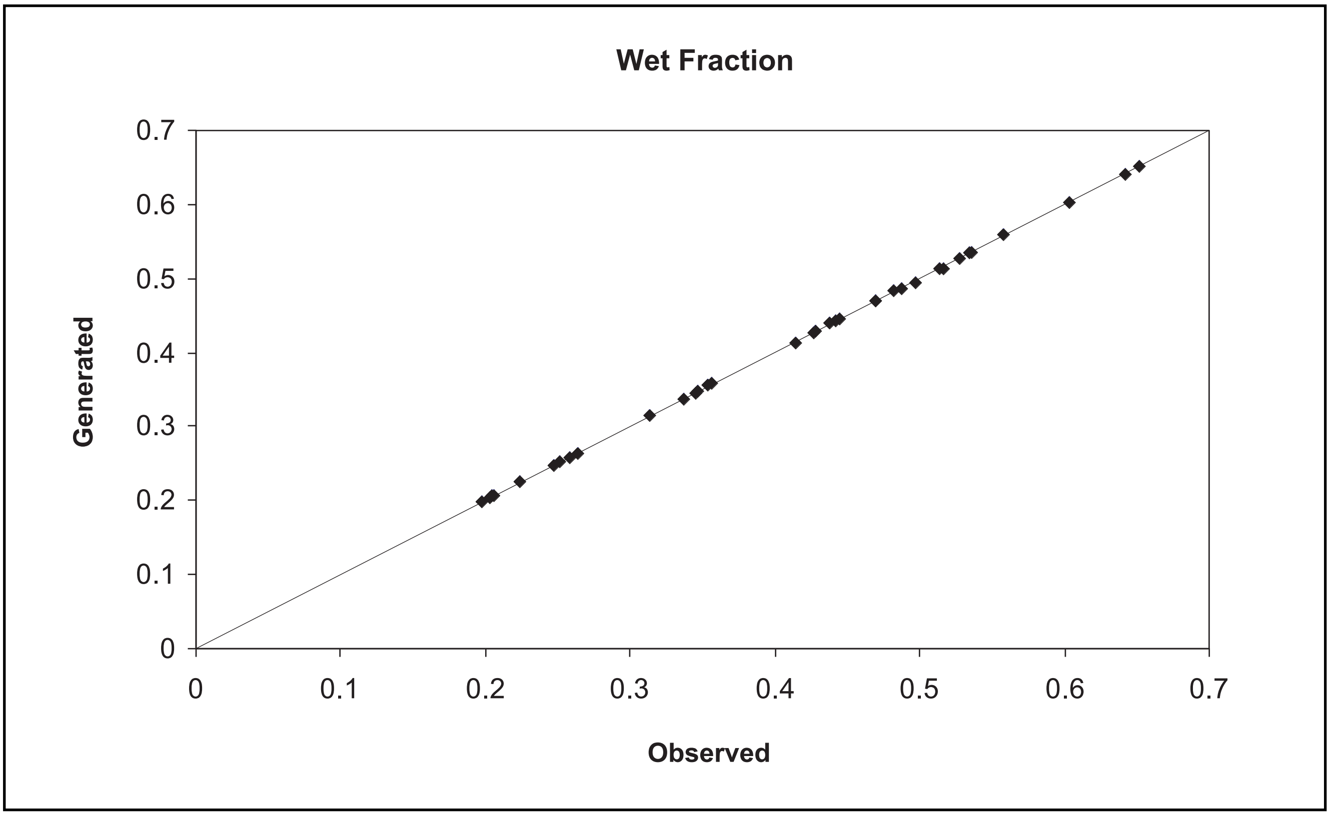

Figure 7. Wet Fraction for the Woady Yaloak Catchment (3 sites x 12 months).

Table 6. Comparison of Historical and Generated Annual Parameters for the Woady Yaloak Catchment.

Parameter		089002	089024	089025
Mean	**Hist**	702	569	623
	Gen	704	570	623
Std Dev	**Hist**	132	109	109
	Gen	133	109	109
Skew	**Hist**	-0.02	0.01	-0.11
	Gen	0.29	0.28	0.28
Correlation	**Hist**	-0.12	-0.03	0.01
	Gen	-0.14	-0.05	-0.01
Maximum	**Hist**	996	841	885
	Gen	1060	865	916
Minimum	**Hist**	343	265	301
	Gen	419	331	387
Range	**Hist**	1530	949	769
	Gen	1229	1068	1129
2-year low sum	**Hist**	1057	830	899
	Gen	1019	799	897
5-year low sum	**Hist**	2977	2371	2588
	Gen	2938	2348	2596
7-year low sum	**Hist**	4279	3587	3735
	Gen	4265	3423	3763
10-year low sum	**Hist**	6230	5286	5780
	Gen	6305	5061	5558

5.2 Yarra River Catchment

The log-odds ratio shown in Figure 8 shows that the spatial correlation between the rainfall occurrence processes at the 20 sites is preserved well. Also, the wet fraction is preserved well (Figure 9).

The historical and generated annual parameters for the Yarra Catchment are shown in Figure B5 in Appendix B. This shows that the model preserves all the annual parameters except the range well. Both the historical and generated skewness are small and not significantly different from zero. The range has a small spread around the 45 degree line and is reasonably preserved.

The historical and generated monthly parameters are shown in Figure B6. The figure shows that all the monthly parameters are satisfactorily preserved except the skewness. A comparison of historical and generated daily rainfall parameters are shown in Figure B7. The mean number of wet days per month, maximum daily rainfall, mean and standard deviation of daily rainfall, dry and wet spell lengths are satisfactorily preserved. The skewness of daily rainfall and the correlation between rainfall depth and duration of wet spell are slightly underestimated while those of dry and wet spell lengths are not preserved well. Mean rainfall on wet days bounded on one side by a wet day is preserved while the mean on solitary wet days and wet days bounded by wet days on both sides are not preserved well.

The cross correlations between the rainfall occurrences at different sites are preserved well (Figure B8). The cross correlations between daily rainfall amounts are preserved well while those between monthly and annual rainfall are underestimated.

5.3 Murrumbidgee River Catchment

The log-odds ratio shown in Figure 10 shows that the spatial correlation between the rainfall occurrence processes at the 30 sites in the Murrumbidgee catchment is preserved well. Also, the wet fraction is also preserved well (Figure 11).

The historical and generated annual parameters for the Murrumbidgee Catchment are shown in Figure B9 in Appendix B. This shows that the model preserves all the annual parameters except the skewness and range well. Both the skewness and range are underestimated.

The historical and generated monthly parameters are shown in Figure B10. The figure shows that all the monthly parameters except skewness and minimum are satisfactorily preserved. The smaller skewness values are preserved and a slight underestimation is evident for larger skewness values. The minimum values generated are smaller than the corresponding historical values and is not considered as a serious drawback.

A comparison of historical and generated daily rainfall parameters are shown in Figure B11. The mean number of wet days per month, maximum daily rainfall, mean and standard deviation of daily rainfall, dry and wet spell lengths are satisfactorily preserved. The skewness of daily rainfall and the correlation between rainfall depth and duration of wet spell are slightly underestimated while those of dry and wet spell lengths are not preserved well. Mean rainfall on wet days bounded on one side by a wet day is preserved while the mean on solitary wet days and wet days bounded by wet days on both sides are not preserved well.

The cross correlations between the rainfall occurrences at different sites are preserved well (Figure B12). The cross correlations between daily rainfall amounts are preserved well while those between monthly and annual rainfall are underestimated.

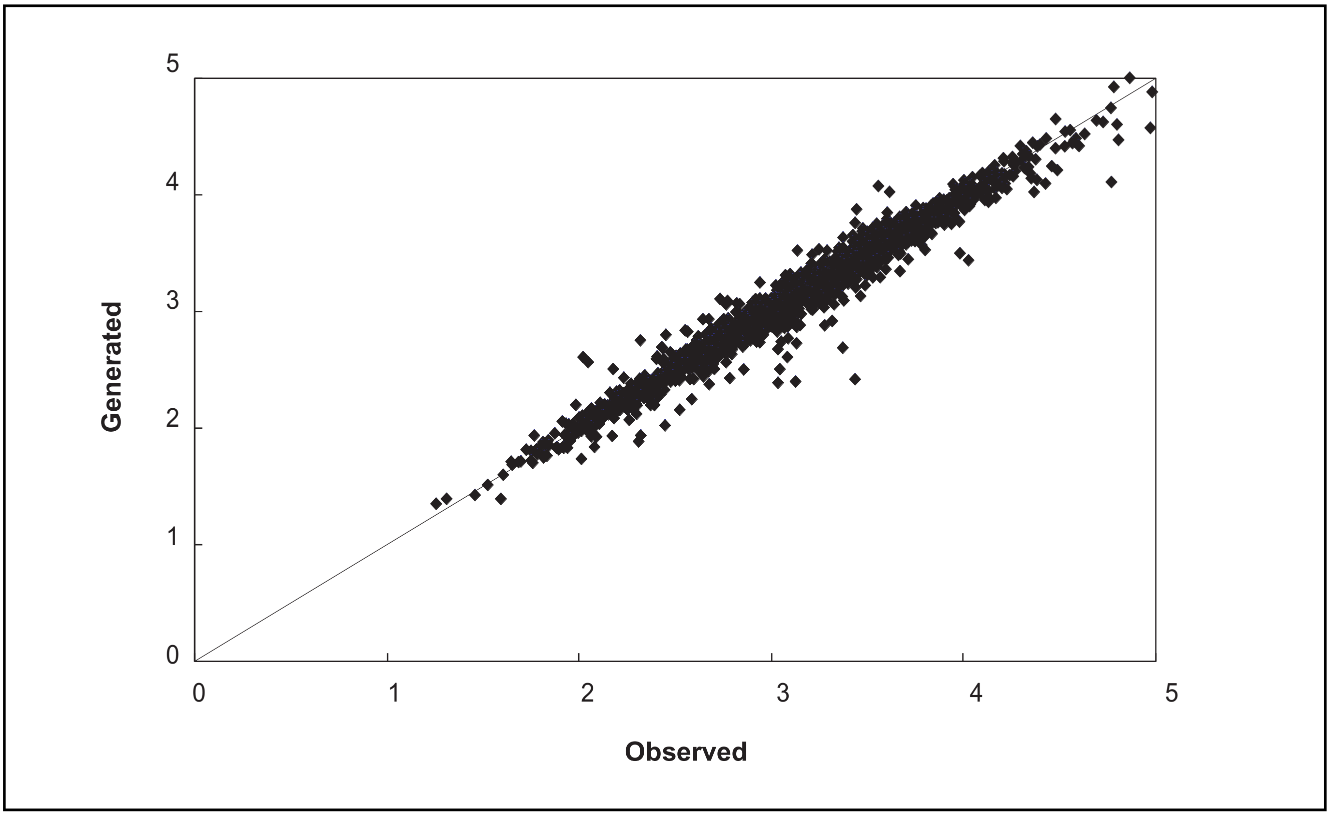

Figure 8. Comparison of Log-odds Ratio for the Yarra Catchment (45 inter-site x 12 months).

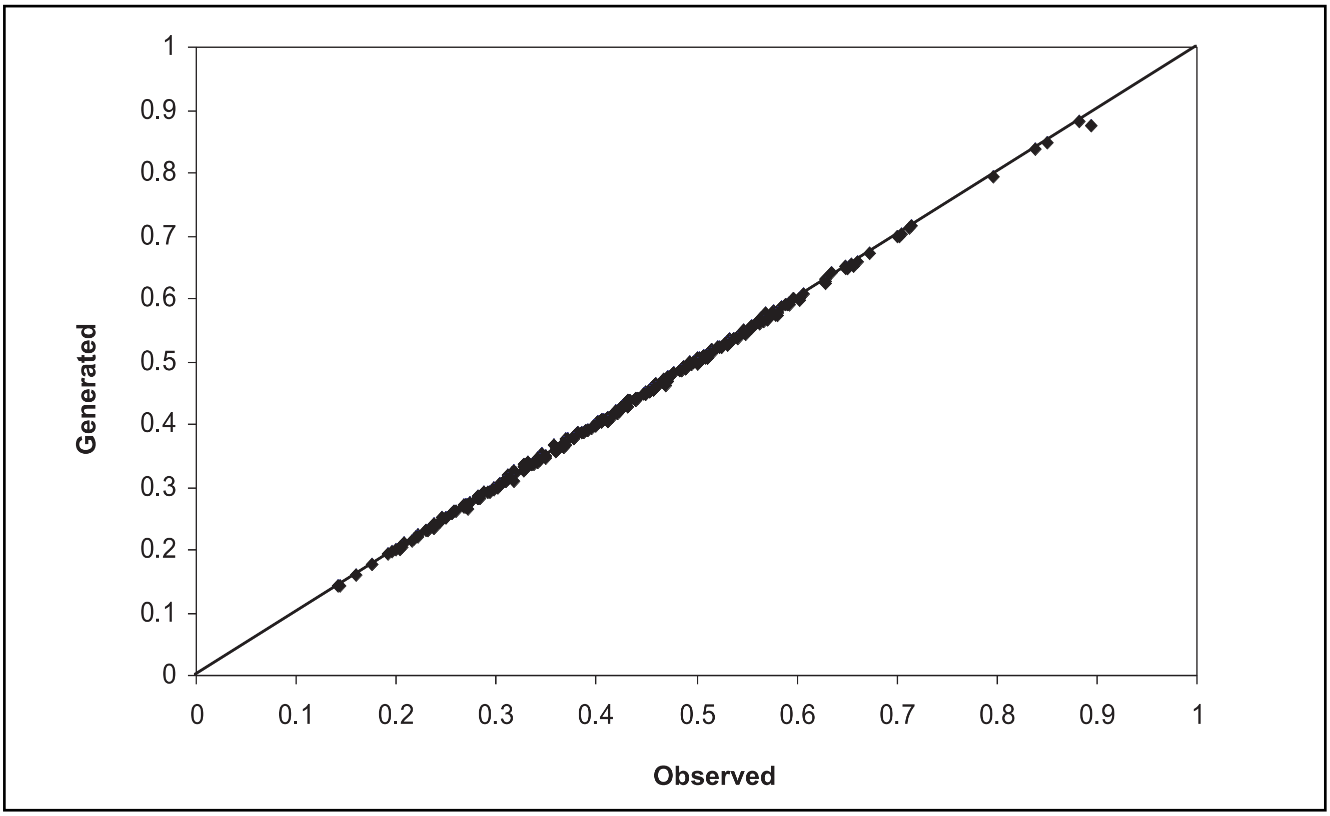

Figure 9. Comparison of Wet Fraction for the Yarra Catchment (10 sites x 12 months).

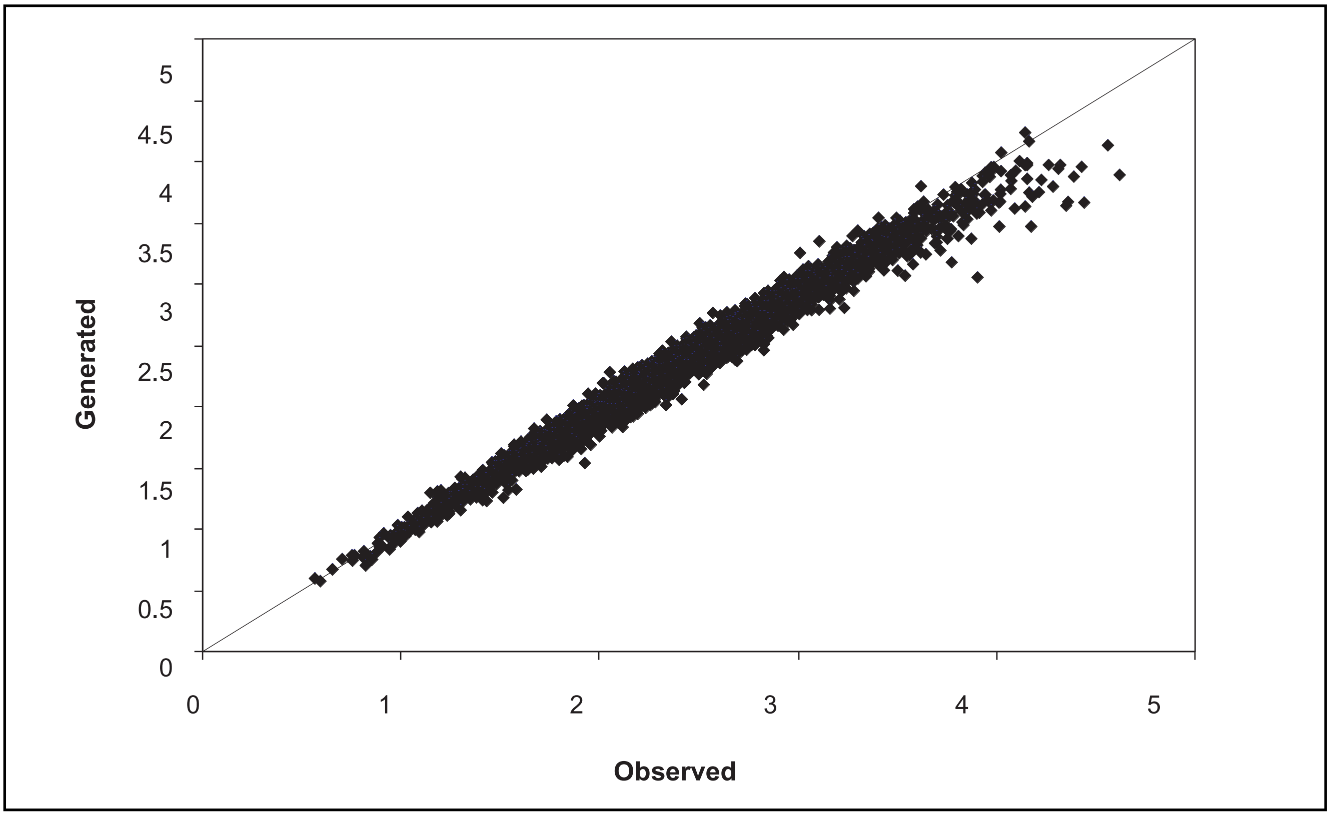

Figure 10. Log-odds Ratio for the Murrumbidgee Catchment (435 inter-site x 12 months).

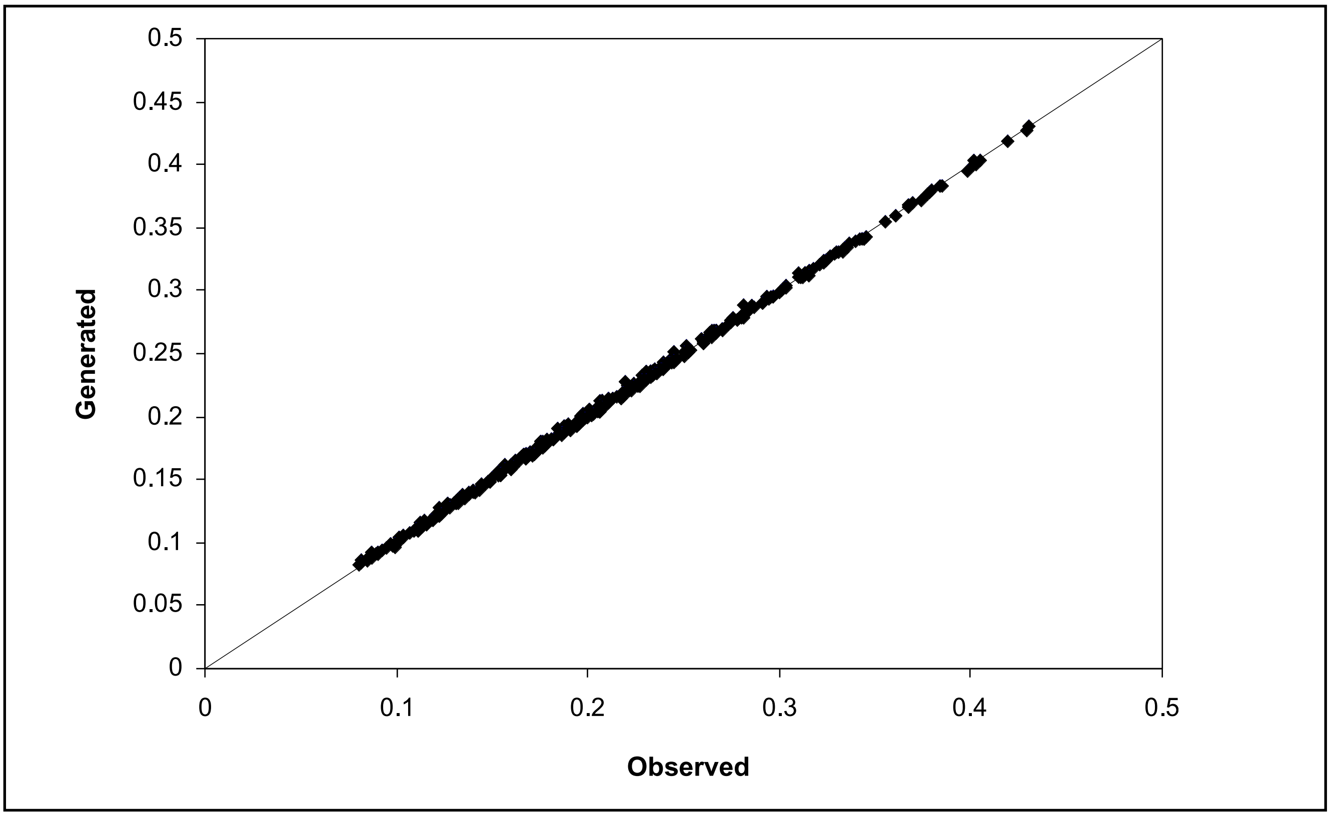

Figure 11. Comparison of Wet Fraction for the Murrumbidgee Catchment (30 sites x 12 months).

5.4 Goulburn-Broken River Catchment

The log-odds ratio shown in Figure 12 shows that the spatial correlation between the rainfall occurrence processes at the four catchments is preserved well. Also, the wet fraction is also preserved well (Figure 13).

The historical and generated annual parameters for the Goulburn-Broken catchment are shown in Figure B13 in Appendix B. This shows that the model preserves all the annual parameters except the range well. Both the historical and generated skewness are small and not significantly different from zero. There is a small overestimation of the range.

The historical and generated monthly parameters are shown in Figure B14. The figure shows that all the monthly parameters except skewness are satisfactorily preserved. A slight underestimation of larger skewness is observed.

A comparison of historical and generated daily rainfall parameters are shown in Figure B15. The mean number of wet days per month, mean and standard deviation of daily rainfall, dry and wet spell lengths are satisfactorily preserved. The maximum daily rainfall is slightly overestimated. The skewness of daily rainfall and the correlation between rainfall depth and duration of wet spell are slightly underestimated while those of dry and wet spell lengths are not preserved well. Mean rainfall on different types of wet days are not preserved well.

The cross correlations between the rainfall occurrences at different sites are preserved well (Figure B16). The cross correlations between annual rainfall amounts are preserved well while those between daily and monthly rainfall are slightly underestimated.

5.5 Sydney Region

The log-odds ratio shown in Figure 14 shows that the spatial correlation between the rainfall occurrence processes at the 30 sites is preserved well. Also, the wet fraction is also preserved well (Figure 15)

The historical and generated annual parameters for the Sydney region are shown in Figure B17 in Appendix B. This shows that the model preserves all the annual parameters except the skewness well. The generated sequences appear to have a skewness of about 0.5.

The historical and generated monthly parameters are shown in Figure B18. The figure shows that all the monthly parameters except skewness and minimum are satisfactorily preserved. There is a slight underestimation is observed for larger skewness values.

A comparison of historical and generated daily rainfall parameters are shown in Figure B19. The mean number of wet days per month, maximum daily rainfall, mean and standard deviation of daily rainfall, dry and wet spell lengths are satisfactorily preserved. The skewness of daily rainfall and the correlation between rainfall depth and duration of wet spell are slightly underestimated while those of dry and wet spell lengths are not preserved well. Mean rainfall on wet days bounded on only one side by a wet day is preserved while the mean on solitary wet days and wet days bounded by wet days on both sides are not preserved well.

The cross correlations between the rainfall occurrences at different sites are preserved well (Figure B20). The cross correlations between daily rainfall amounts are preserved well while those between monthly and annual rainfall have a small spread.

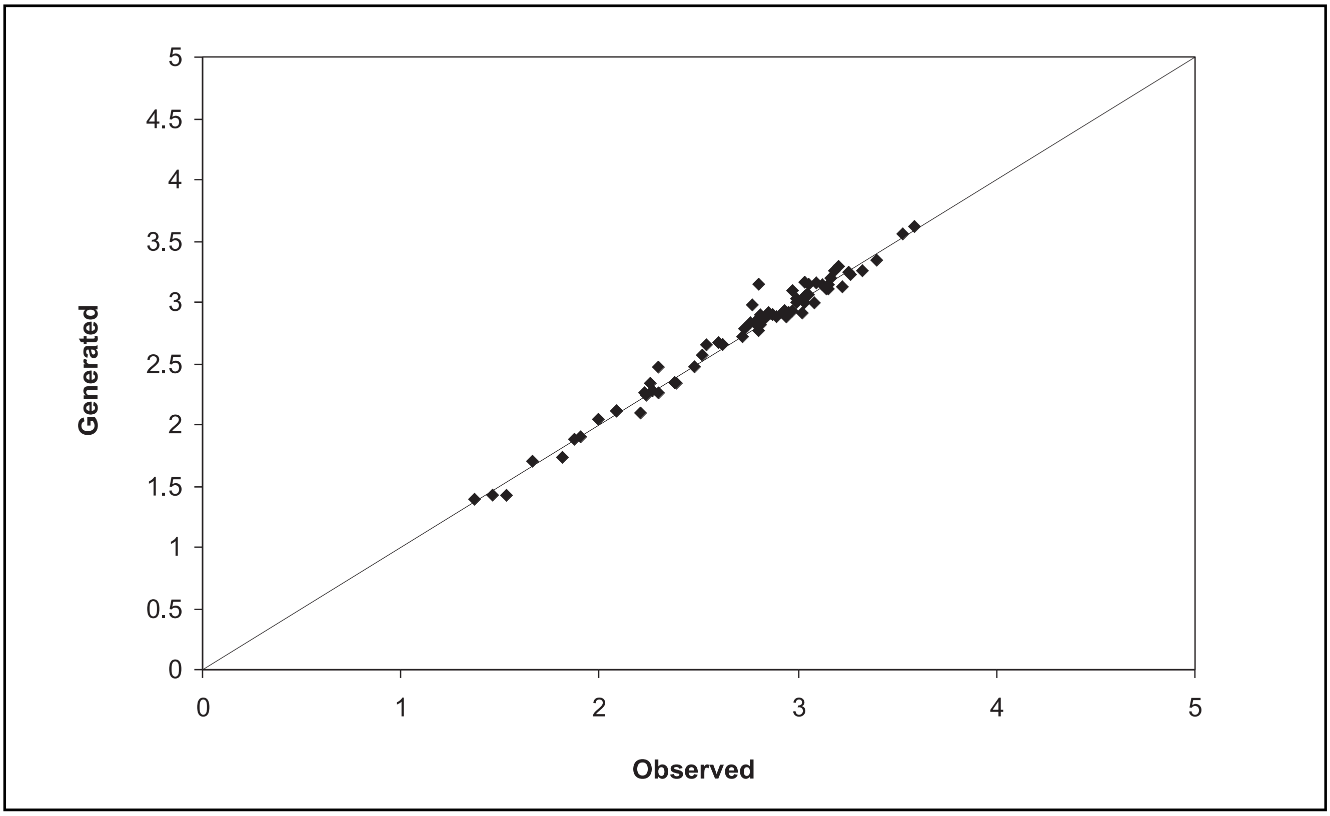

Figure 12. Log-odds Ratio for the Goulburn-Broken Catchment (6 inter-site x 12 months).

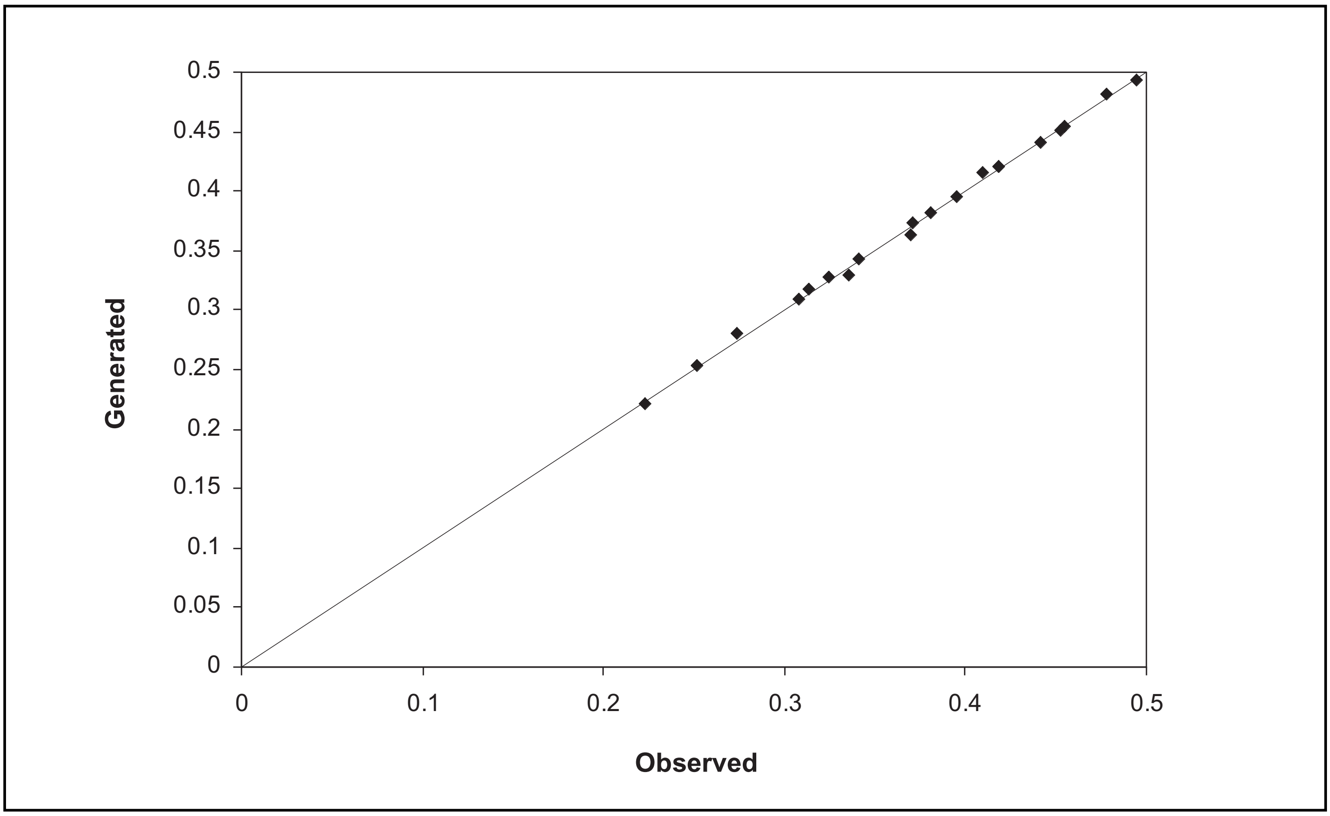

Figure 13. Comparison of Wet Fraction for the Goulburn-Broken Catchment (4 sites x 12 months).

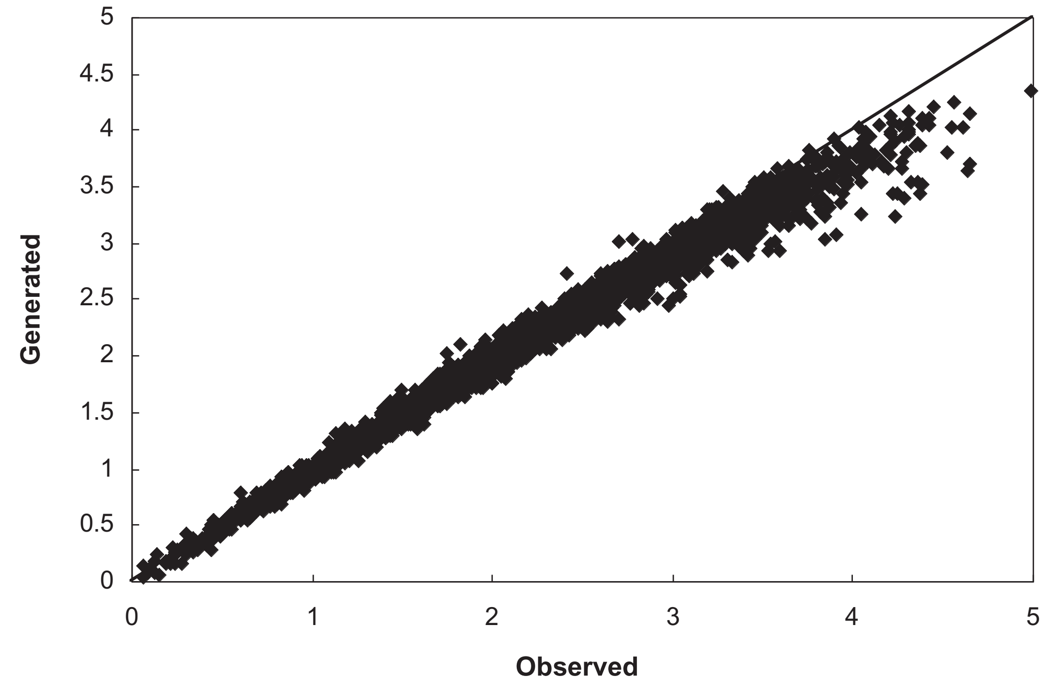

Figure 14. Log-odds Ratio for the Sydney Region (435 inter-site x 12 months).

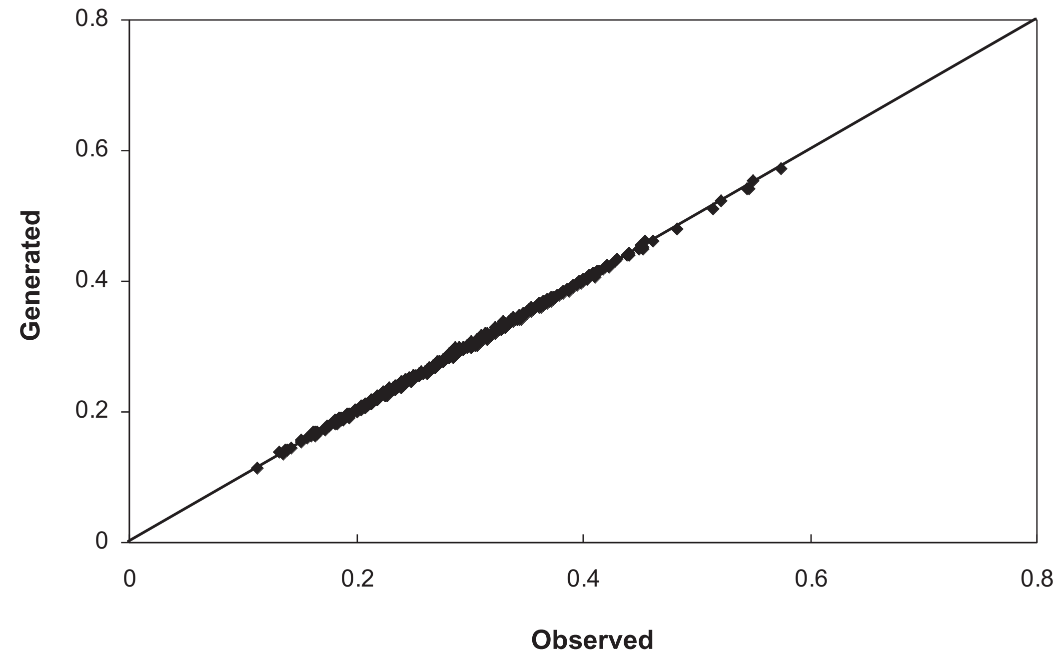

Figure 15. Comparison of Wet Fraction for the Sydney Region (30 sites x 12 months).

6. Conclusions

A multi-site two-part model based on Wilks Model (Wilks 1998) is developed to generate daily rainfall at a number of sites. Daily rainfall occurrence is modelled by a first order Markov chain and the correlation between the rainfall occurrence is handled by using correlated random numbers. A two parameter Gamma distribution is used to generate the rainfall depths and the spatial correlation between rainfall depths is introduced by using correlated random numbers. To preserve the monthly and annual characteristics, the generated daily rainfall at each site is nested within monthly and annual models. The attempt to improve the spatial correlation of monthly and annual rainfall was not successful as the correlated noise terms violated the assumptions of the multi-site model formulation.

The model was applied to five catchments/regions with the number of rainfall sites varying from three to thirty. A comparison of the historical and generated statistics showed that the model preserves all the important characteristics of rainfall at the daily, monthly and annual time scales. Only the skewness of monthly rainfall and the spatial cross correlations at the monthly and annual time scales were not preserved well. The model is considered adequate as it preserves all the important daily parameters including the daily spatial cross correlations.

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Appendix A – Derivation of Mean and Standard Deviation of Annual Rainfall

Let X_1, X_2, ..., X_{12} represent the monthly rainfall and Z represent the annual rainfall.

$$Z = X_1 + X_2 + \ldots + X_{12}$$

$$= \sum_{j=1}^{12} X_j$$

$$E[Z] = \sum_{j=1}^{12} E[X_j]$$

$$(Z) = \sum_{j=1}^{12} \mu(X_j)$$

Without loss of generality, let us standardise the annual and monthly values to have zero mean.

$$z = Z - [Z]$$

$$Var[z] = E[z^2] = Var[Z] = \sigma^2(Z)$$

$$x_j = X_j - [X_j]$$

$$Var[x_j] = E[x_j^2] = Var[X_j] = \sigma^2(X_j)$$

$$E[z^2] = E[(x_1 + x_2 + \ldots + x_{12})^2]$$

$$\approx \sum_{j=1}^{12} E[x_j^2] + 2\sum_{j=2}^{12} E[x_j x_{j-1}] + 2\sum_{j=3}^{12} E[x_j x_{j-2}] + 2\sum_{j=4}^{12} E[x_j x_{j-3}]$$

$$\sigma^2(Z^k) \approx \sum_{j=1}^{12} \sigma^2(X_j^k) + 2\sum_{j=2}^{12} \sigma(X_j^k)\sigma(X_{j-1}^k)\rho_{j,j-1}^k + 2\sum_{j=3}^{12} \sigma(X_j^k)\sigma(X_{j-2}^k)\rho_{j,j-1}^k \rho_{j-1,j-2}^k$$

$$+ \; 2\sum_{j=4}^{12} \sigma(X_j^k)\sigma(X_{j-3}^k)\rho_{j,j-1}^k \rho_{j-1,j-2}^k \rho_{j-2,j-3}^k$$

where:

$\rho_{j,\,j-k}$ is the correlation between the rainfall for months *j* and *j-k*.

Appendix B – Comparison of Historical and Generated Parameters

The following terms are used to refer various parameters in the figures in Appendix B.

Annual Parameters

1.	Mean	Mean annual rainfall (mm)
2.	Std_Dev	Standard deviation of annual rainfall (mm)
3.	Skew	Coefficient of skewness of annual rainfall
4.	Corr	Lag one autocorrelation coefficient of annual rainfall
5.	Max	Maximum annual rainfall (mm)
6.	Min	Minimum annual rainfall (mm)
7.	Range	Adjusted range (mm)
8.	2-year	Low 2-year rainfall sum
9.	5-year	Low 5-year rainfall sum
10.	7-year	Low 7-year rainfall sum
11.	10-year	Low 10-year rainfall sum
12.	#_wet	Mean annual number of wet days

Monthly Parameters

1.	Mean	Mean monthly rainfall (mm)
2.	Std_Dev	Standard deviation of monthly rainfall (mm)
3.	Skew	Coefficient of skewness of monthly rainfall
4.	Corr	Lag one autocorrelation coefficient of monthly rainfall
5.	Max	Maximum monthly rainfall (mm)
6.	Min	Minimum monthly rainfall (mm)

Daily Parameters

1.	#_Wet	Mean number of wet days in a month
2.	Max	Maximum daily rainfall (mm) in a month
3.	Mean	Mean daily rainfall (mm) for a month
4.	Std_Dev	Standard deviation of daily rainfall (mm) for a month
5.	Skew	Coefficient of skewness of daily rainfall for a month
6.	Wet_0	Mean daily rainfall (mm) on solitary wet days for a month
7.	Wet_1	Mean daily rainfall (mm) on days bounded by wet days on one side for a month
8.	Wet_2	Mean daily rainfall (mm) on days bounded by wet days on both sides for a month
9.	Corr	Correlation between rainfall depth and duration
10.	dsMean	Mean dry spell length (days)
11.	dsSD	Standard deviation of dry spell length
12.	dsSkew	Coefficient of skewness of dry spell length
13.	wsMean	Mean wet spell length (days)
14.	wsSD	Standard deviation of wet spell length
15.	wsSkew	Coefficient of skewness of wet spell length
16.	DS_Max	Maximum dry spell length
17.	WS_Max	Maximum wet spell length

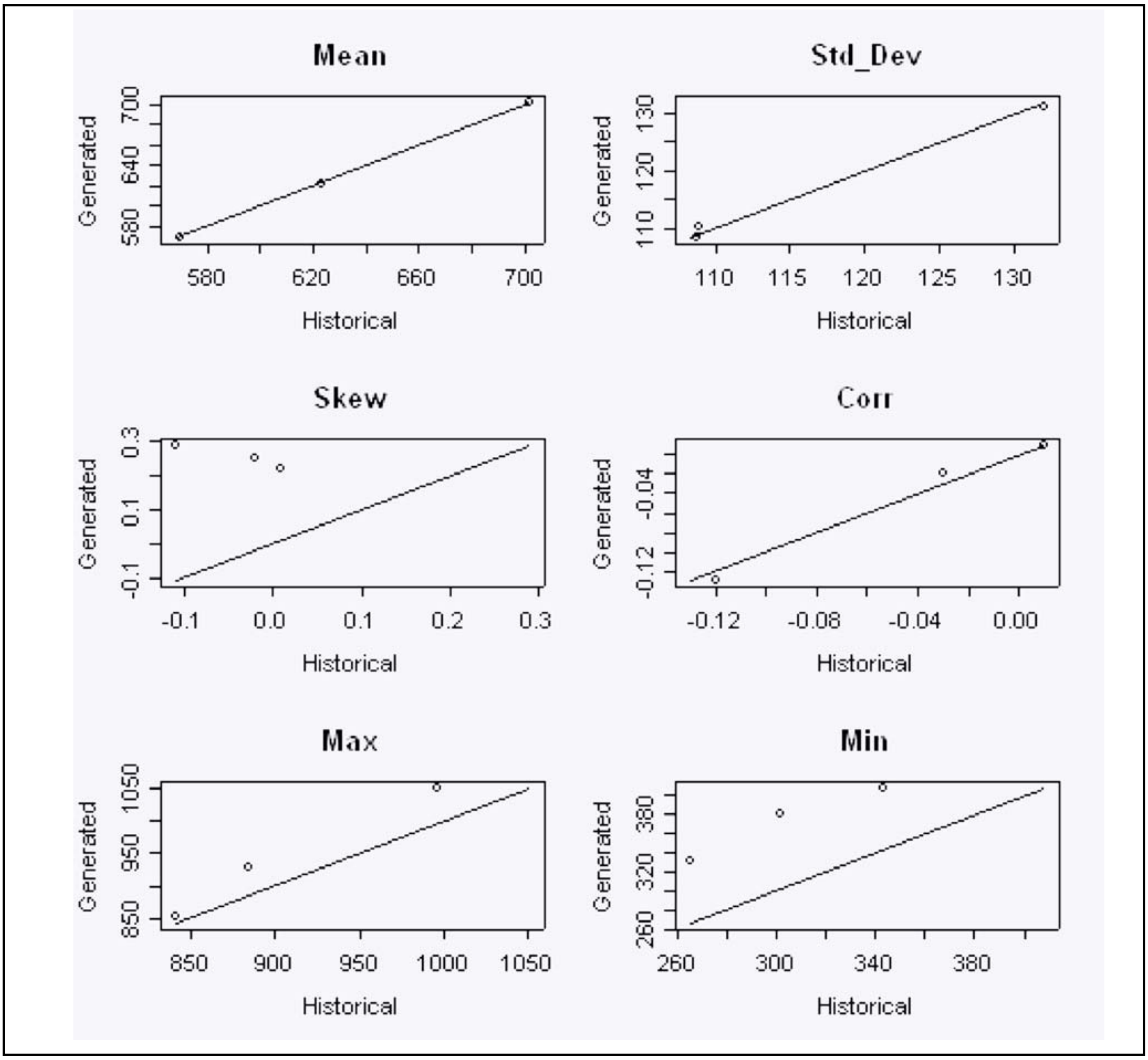

Figure B1. Comparison of Historical and Generated Annual Parameters for Woady Yaloak Catchment.

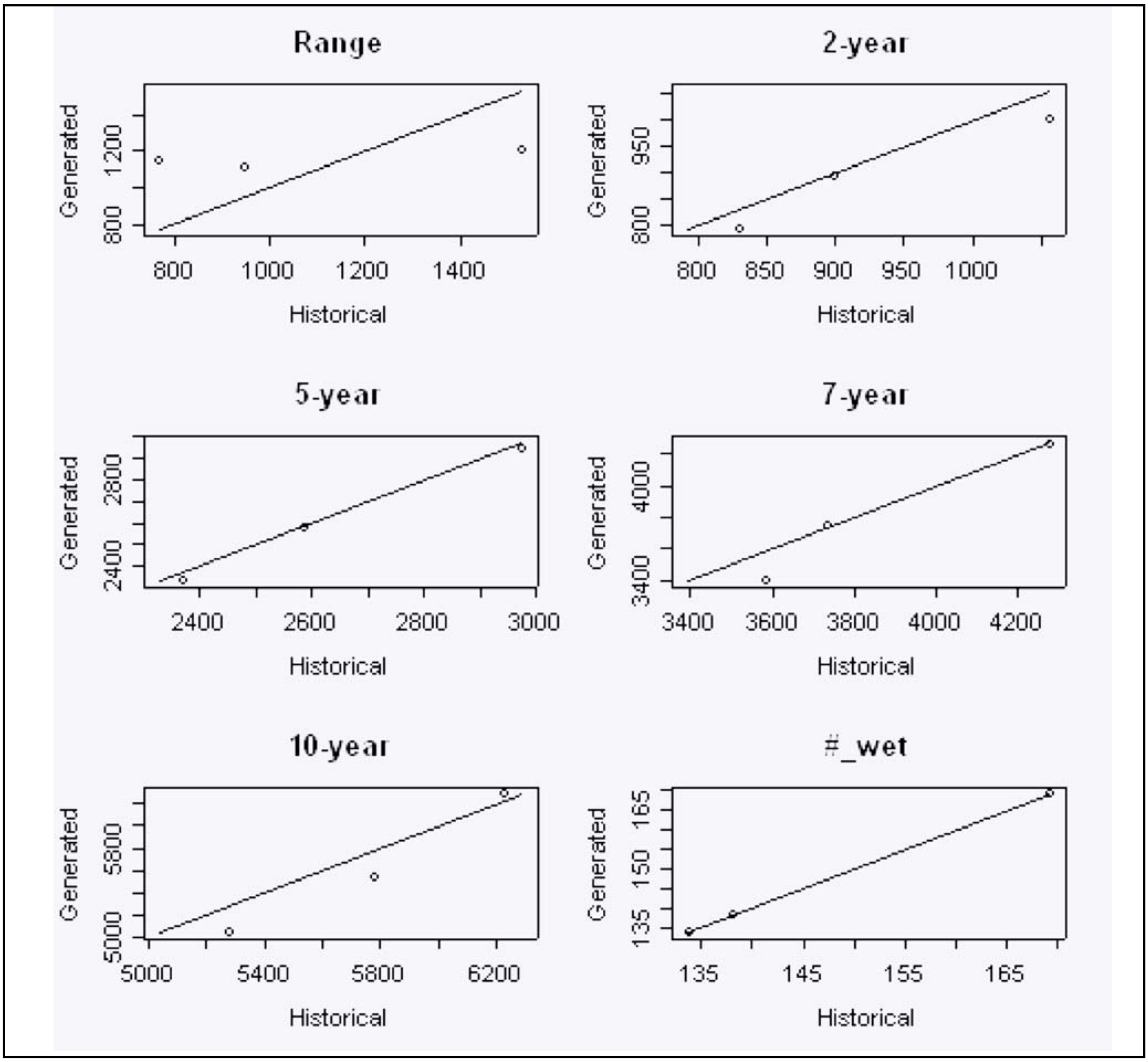

Figure B1. Comparison of Historical and Generated Annual Parameters for Woady Yaloak Catchment. (Cont.)

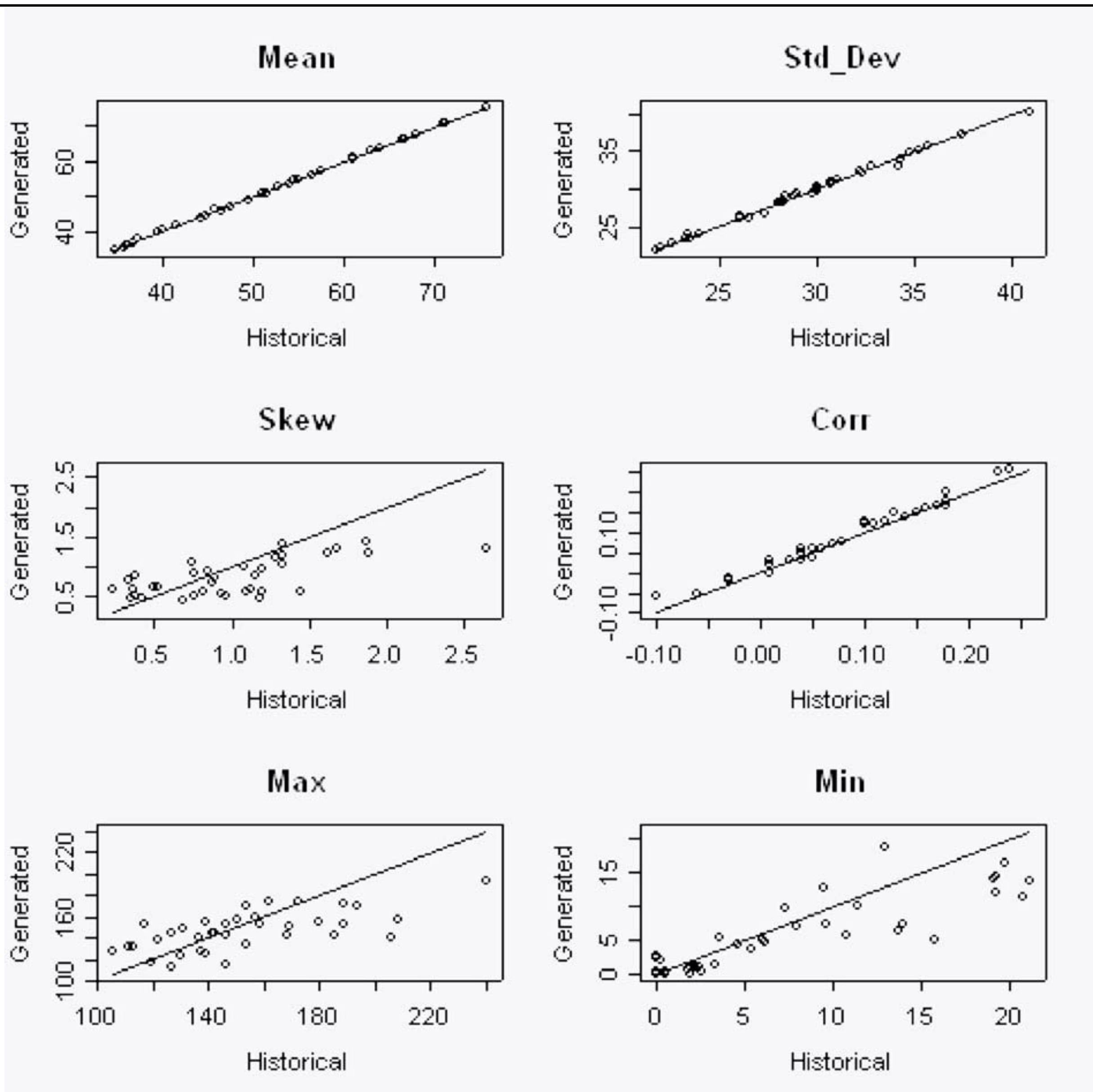

Figure B2. Comparison of Historical and Generated Monthly Parameters for the Woady Yaloak Catchment.

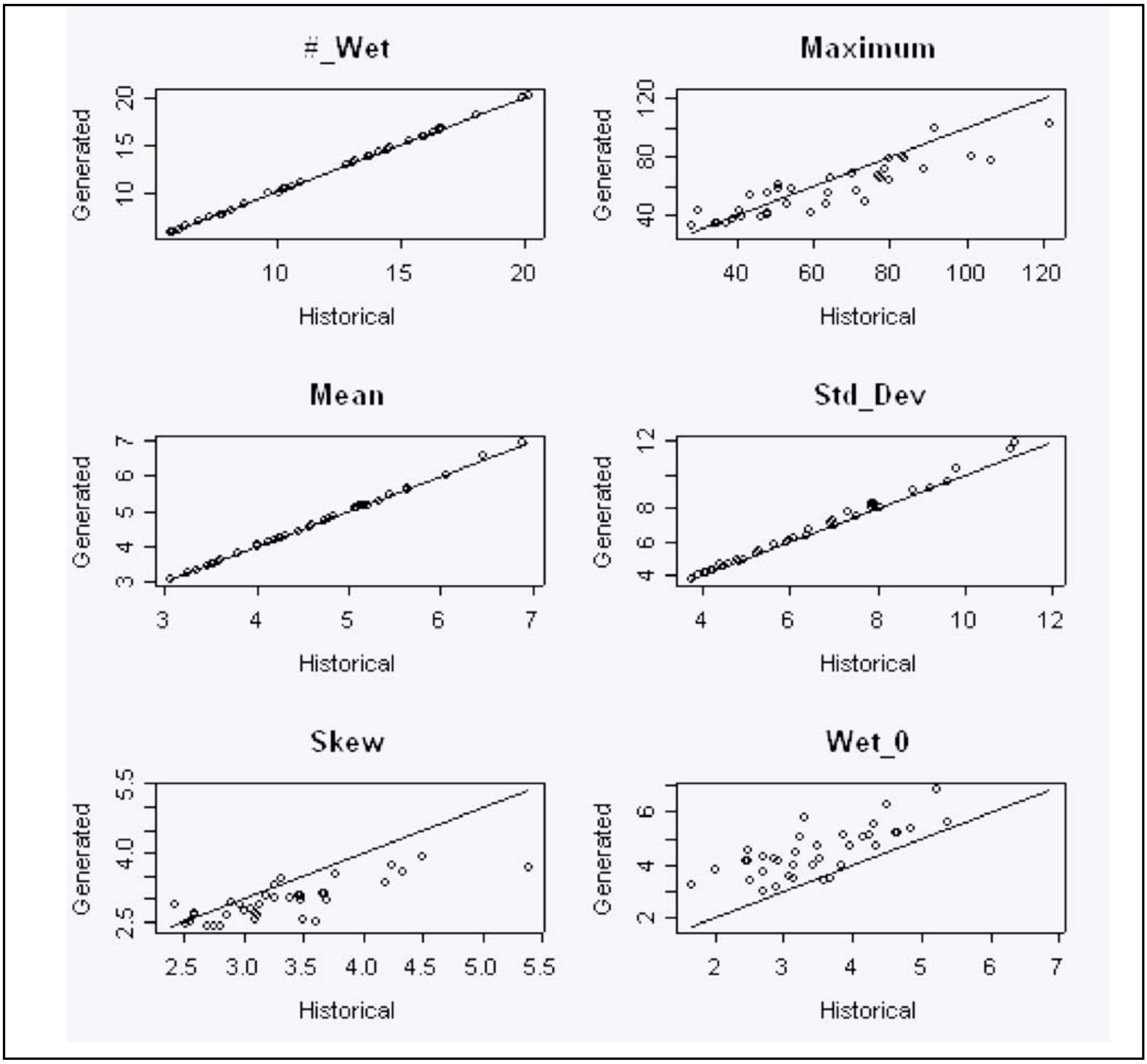

Figure B3. Comparison of Historical and Generated Daily Parameters for the Woady Yaloak Catchment.

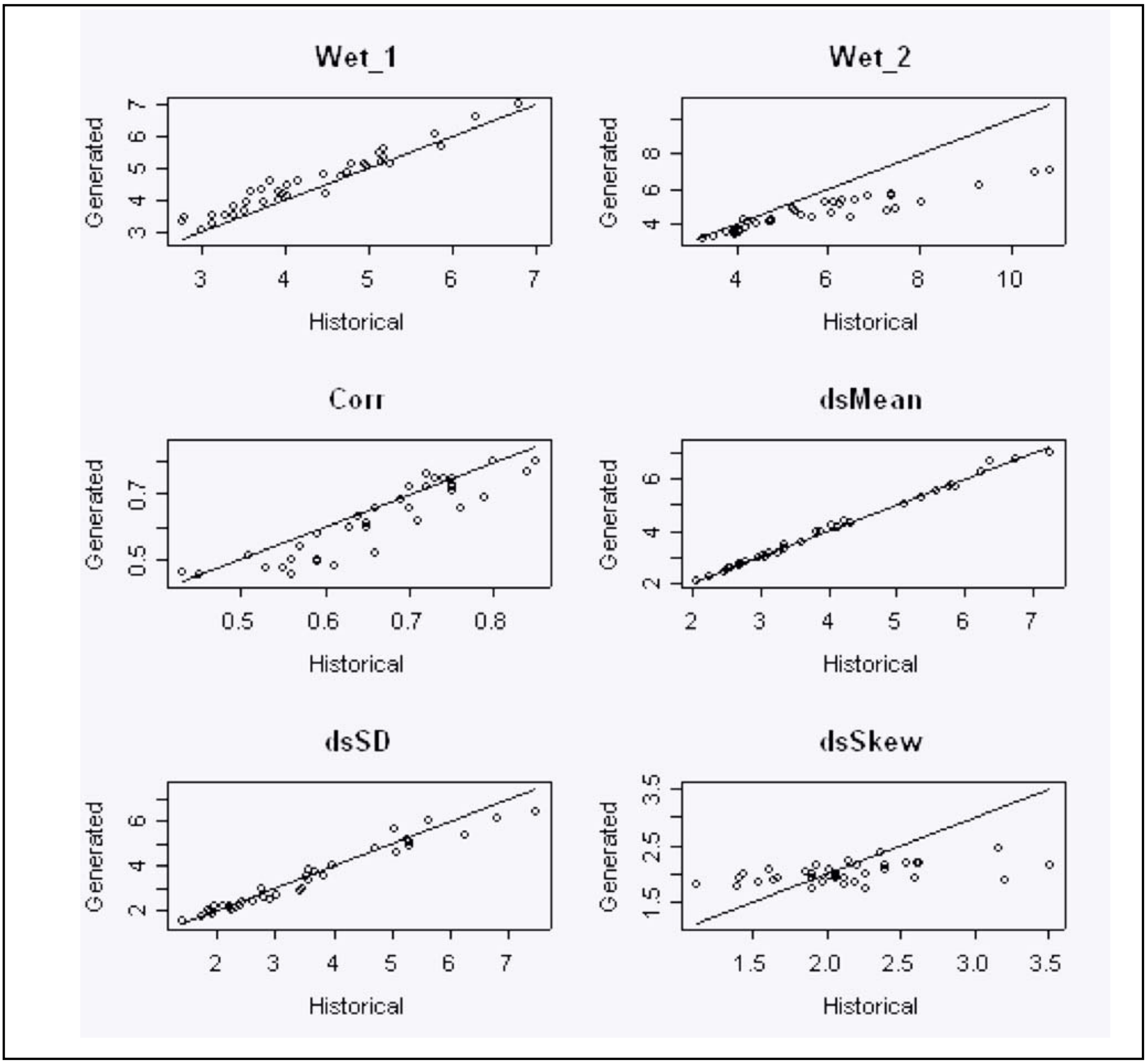

Figure B3. Comparison of Historical and Generated Daily Parameters for the Woady Yaloak Catchment. (Cont.)

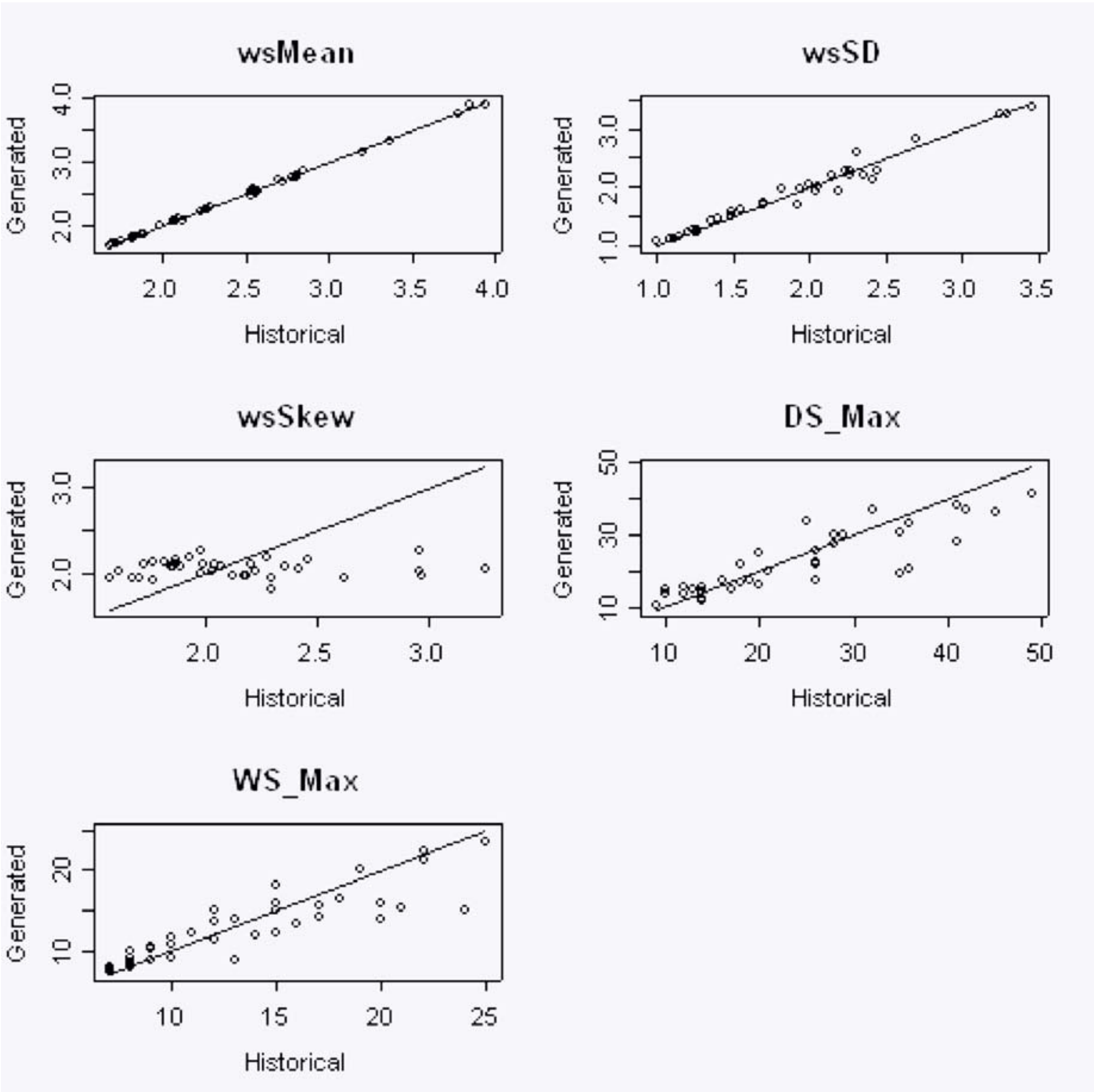

Figure B3. Comparison of Historical and Generated Daily Parameters for the Woady Yaloak Catchment. (Cont.)

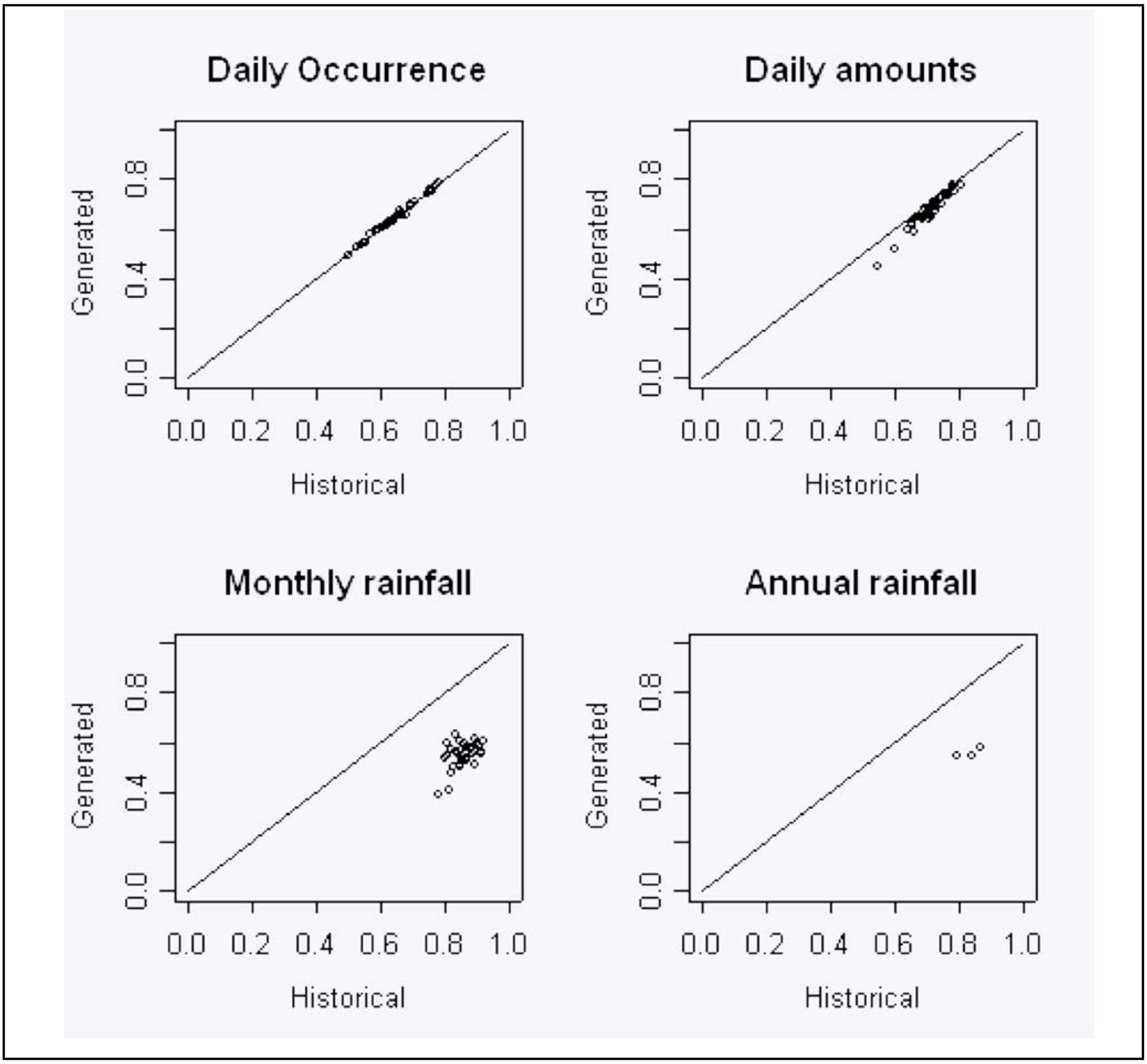

Figure B4. Comparison of Historical and Generated Cross Correlations for the Woady Yaloak Catchment.

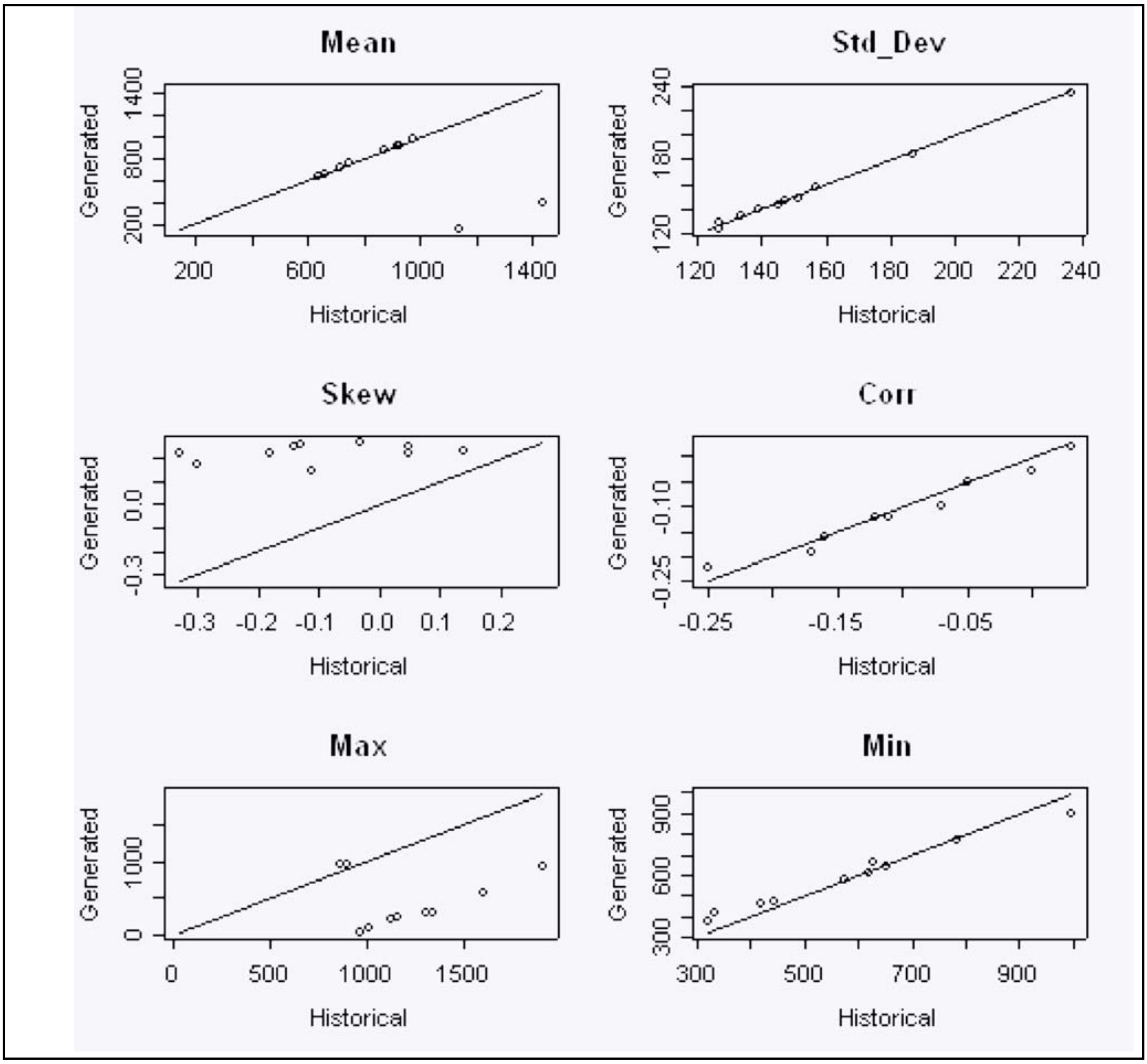

Figure B5. Comparison of Historical and Generated Annual Parameters for the Yarra Catchment.

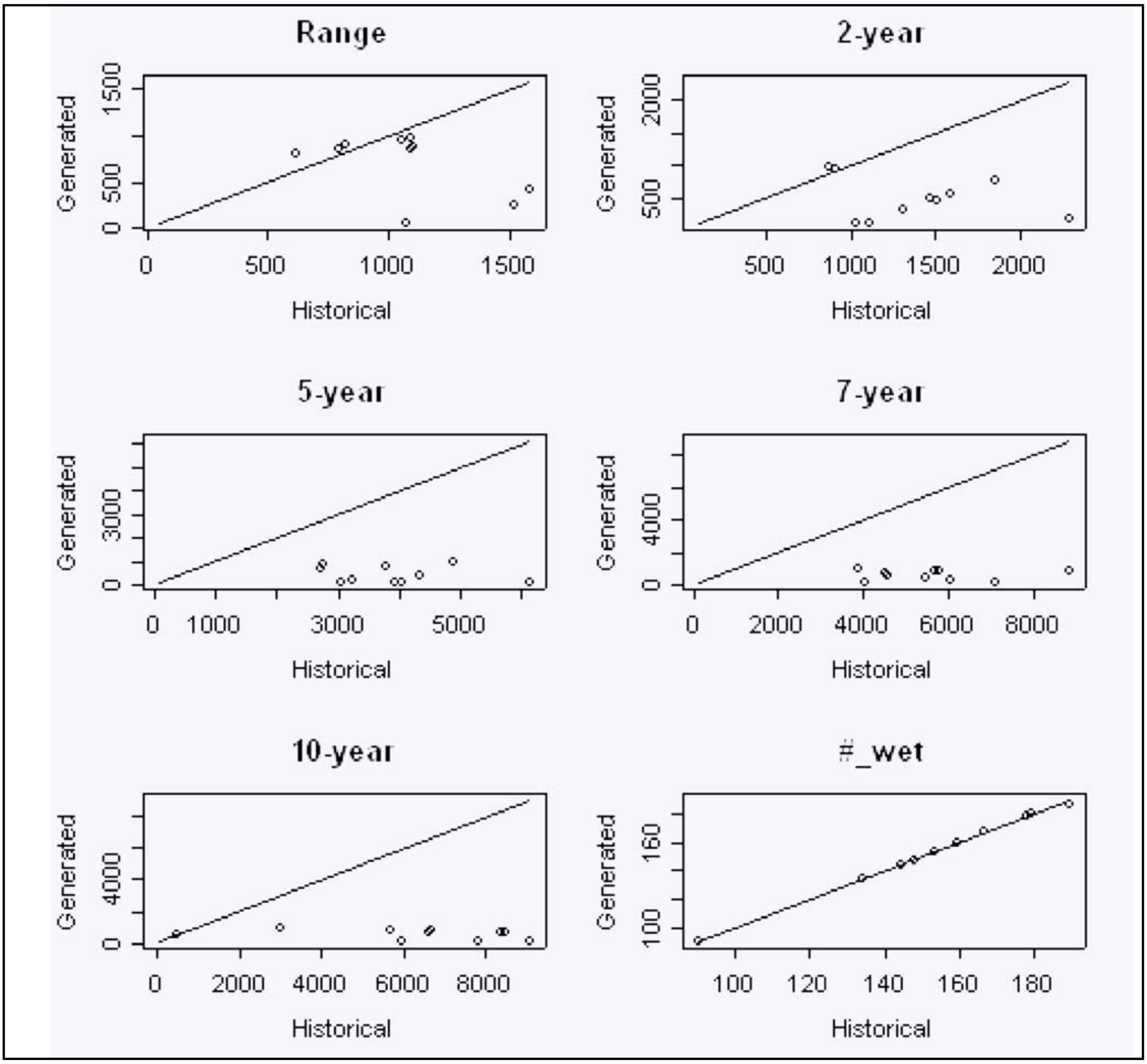

Figure B5. Comparison of Historical and Generated Annual Parameters for the Yarra Catchment. (Cont.)

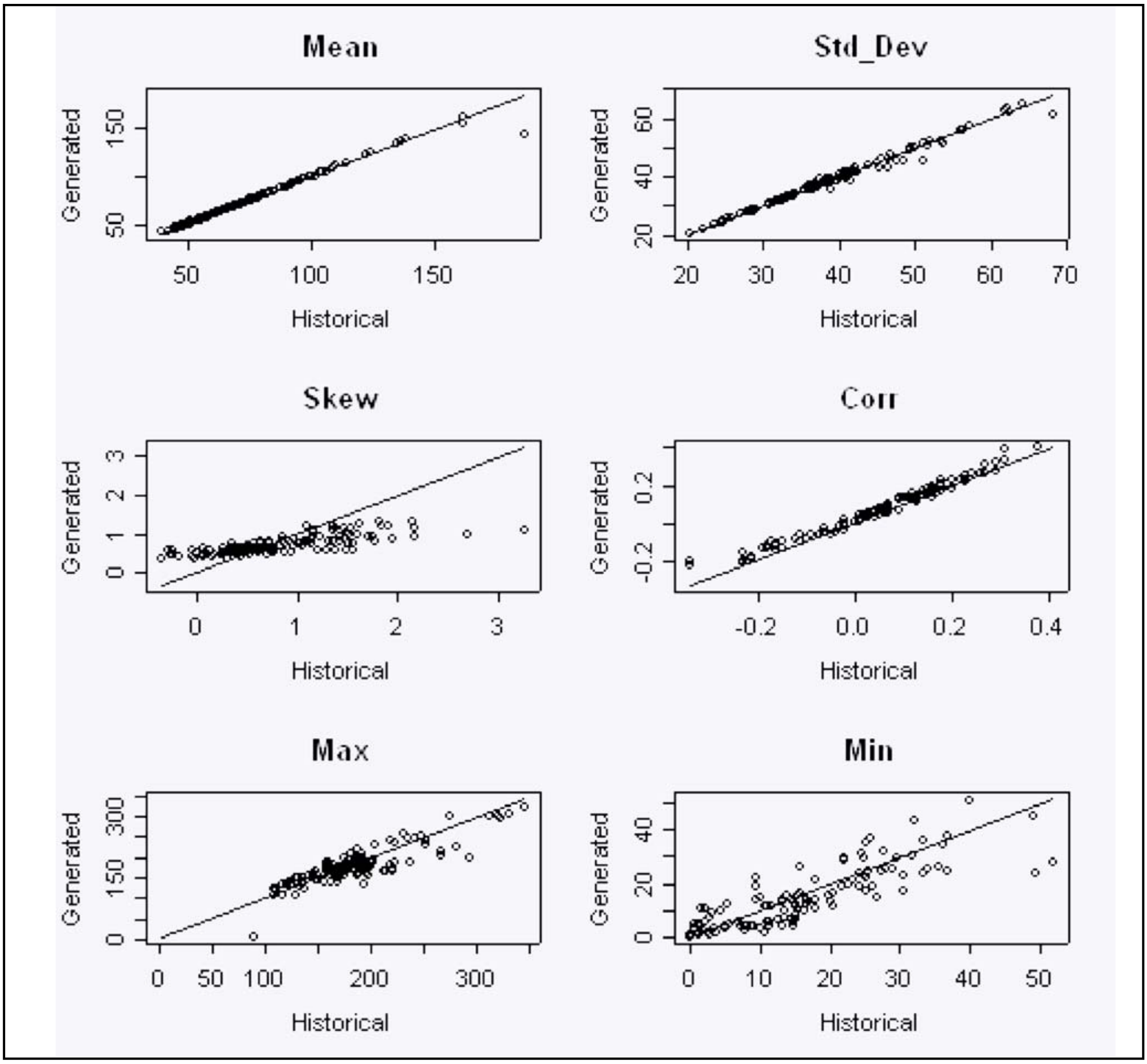

Figure B6. Comparison of Historical and Generated Monthly Parameters for the Yarra Catchment.

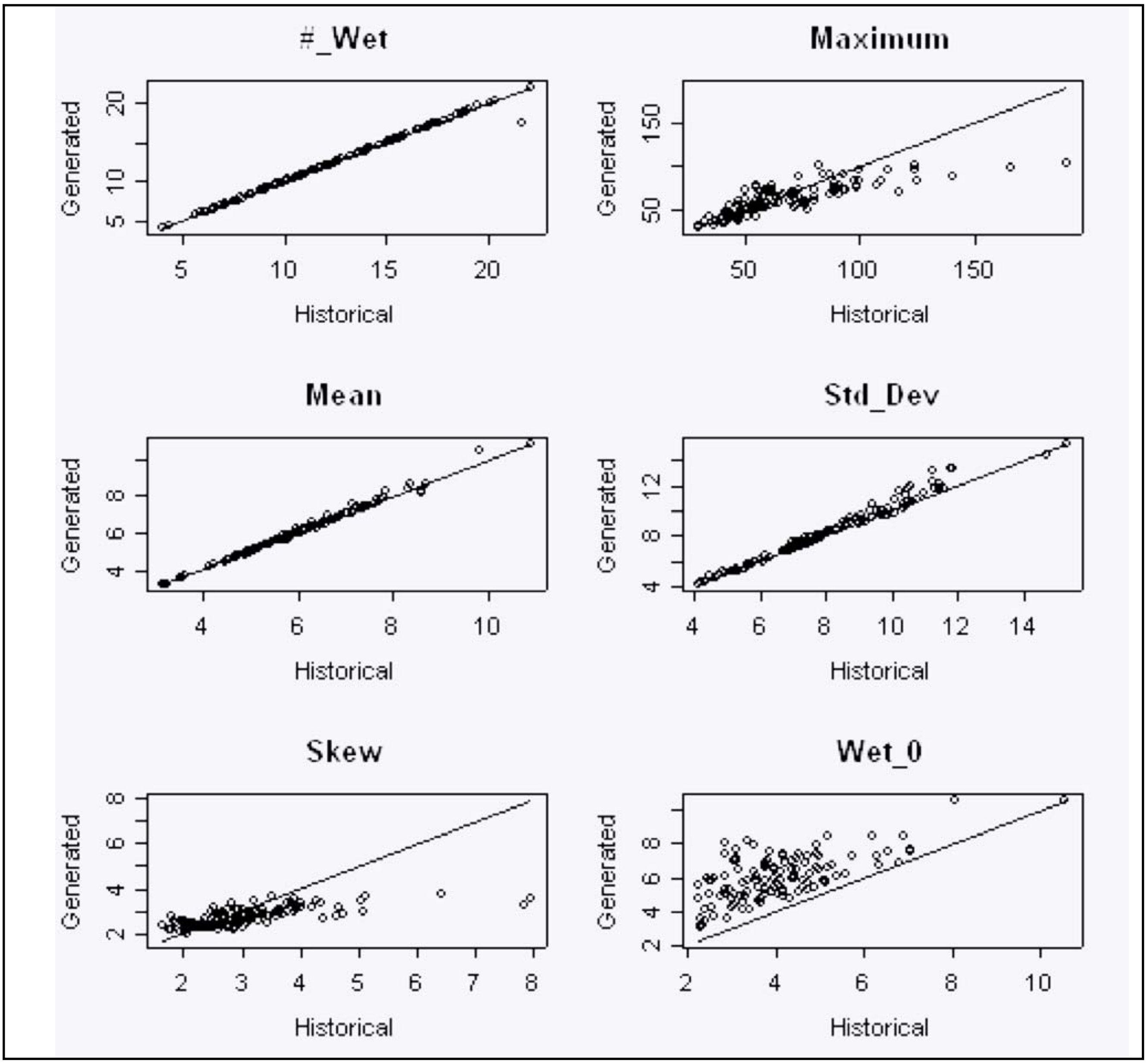

Figure B7. Comparison of Historical and Generated Daily Parameters for the Yarra Catchment.

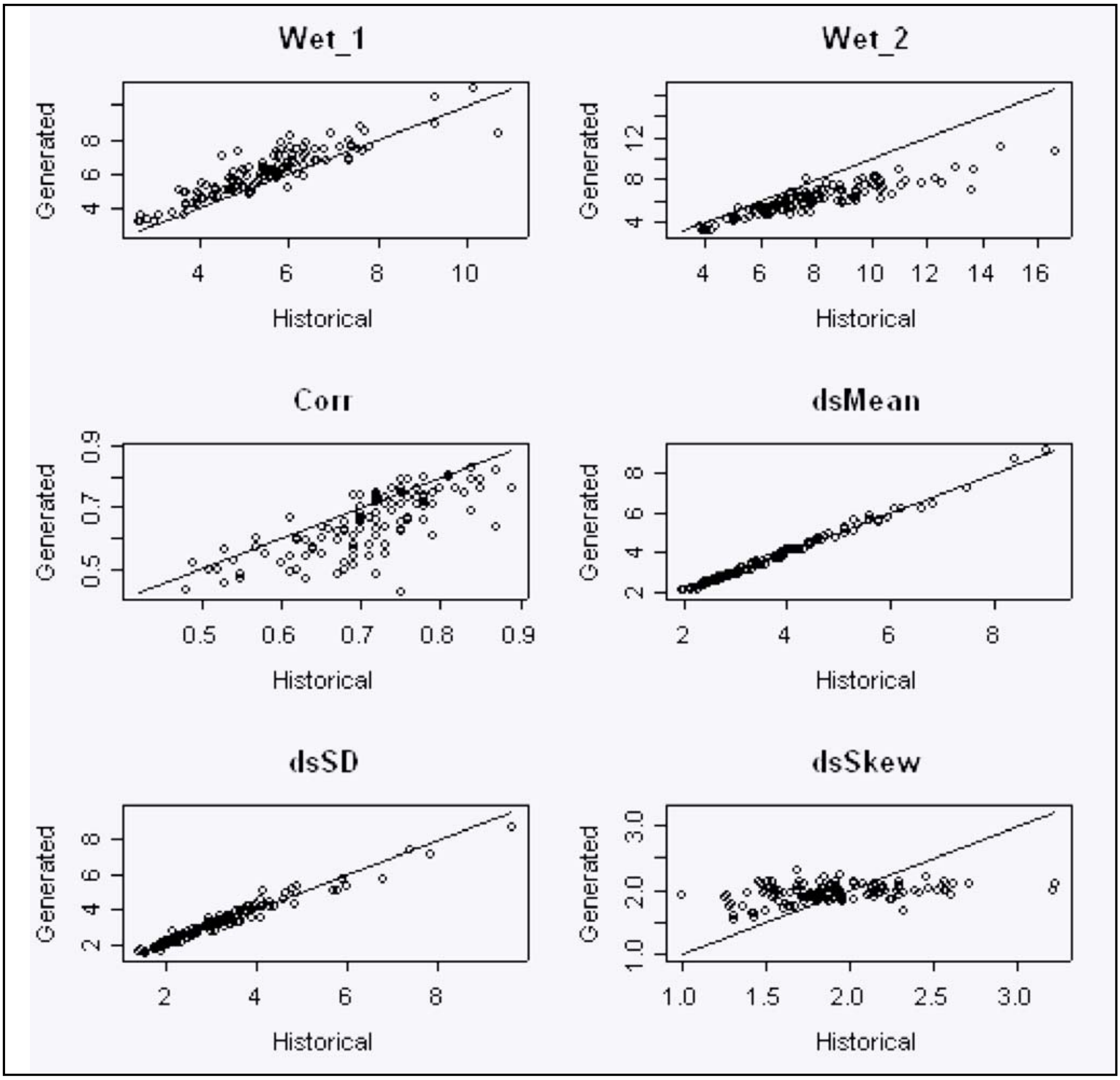

Figure B7. Comparison of Historical and Generated Daily Parameters for the Yarra Catchment. (Cont.)

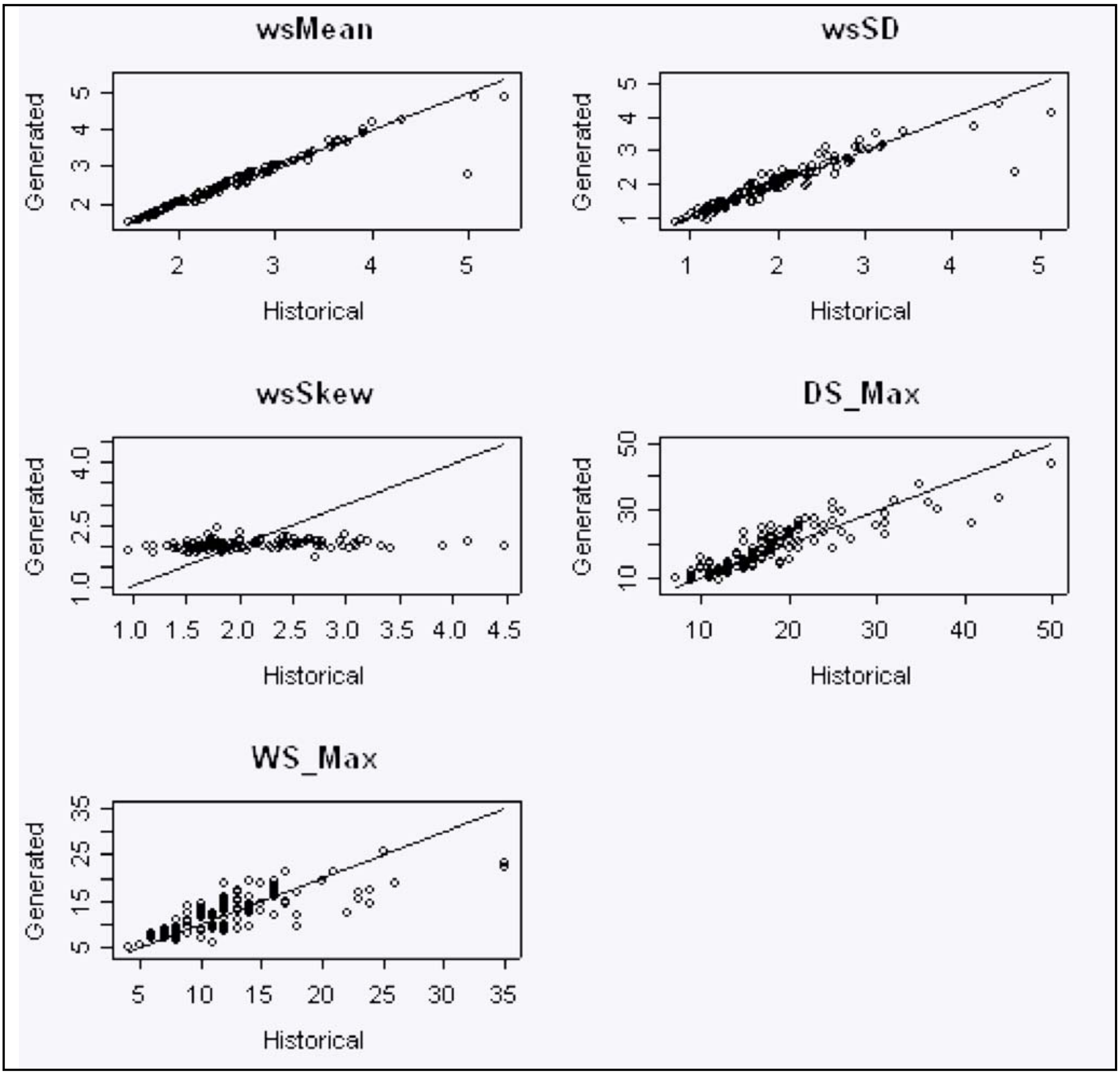

Figure B7. Comparison of Historical and Generated Daily Parameters for the Yarra Catchment. (Cont.)

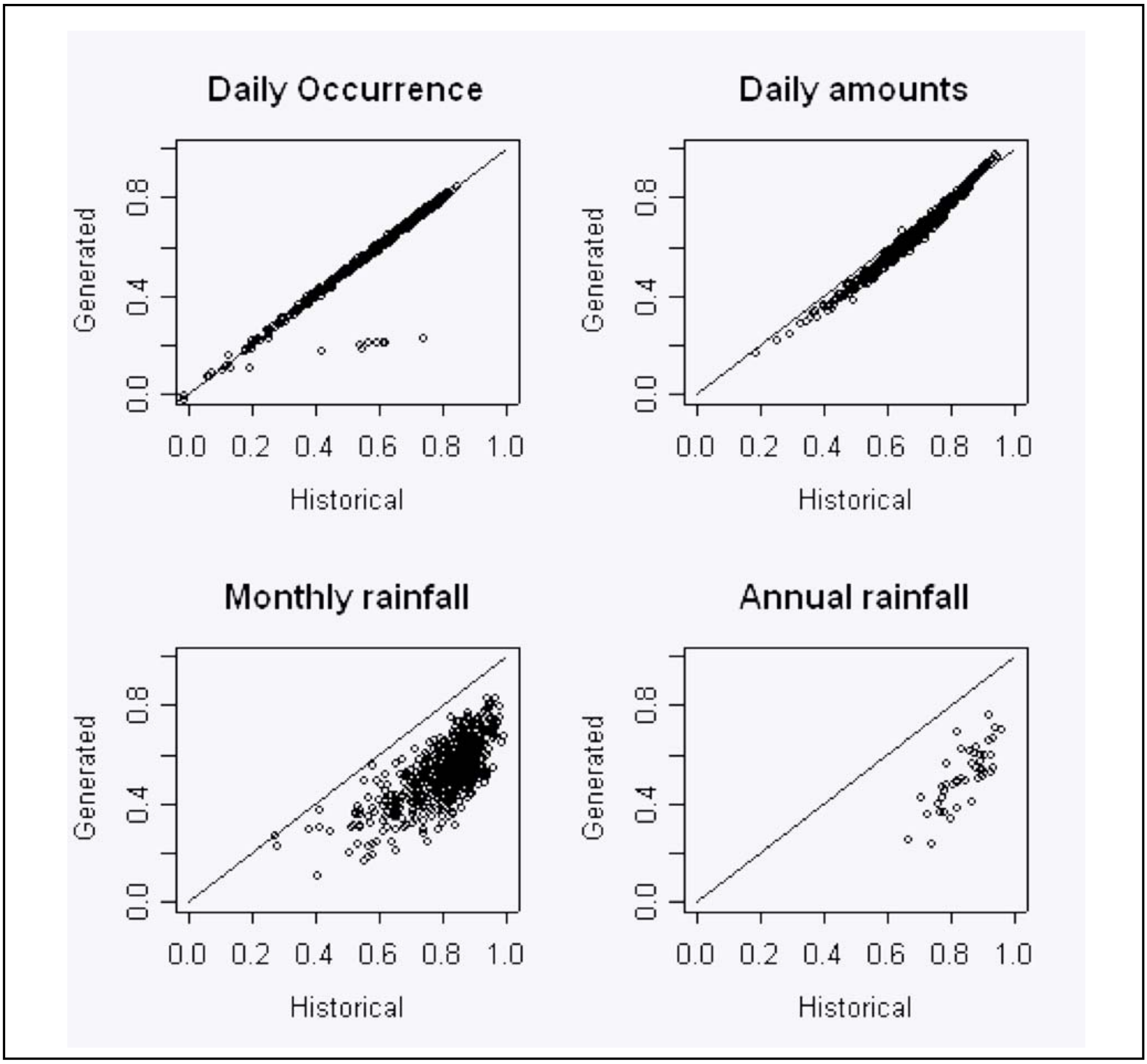

Figure B8. Comparison of Historical and Generated Cross Correlations for the Yarra Catchment.

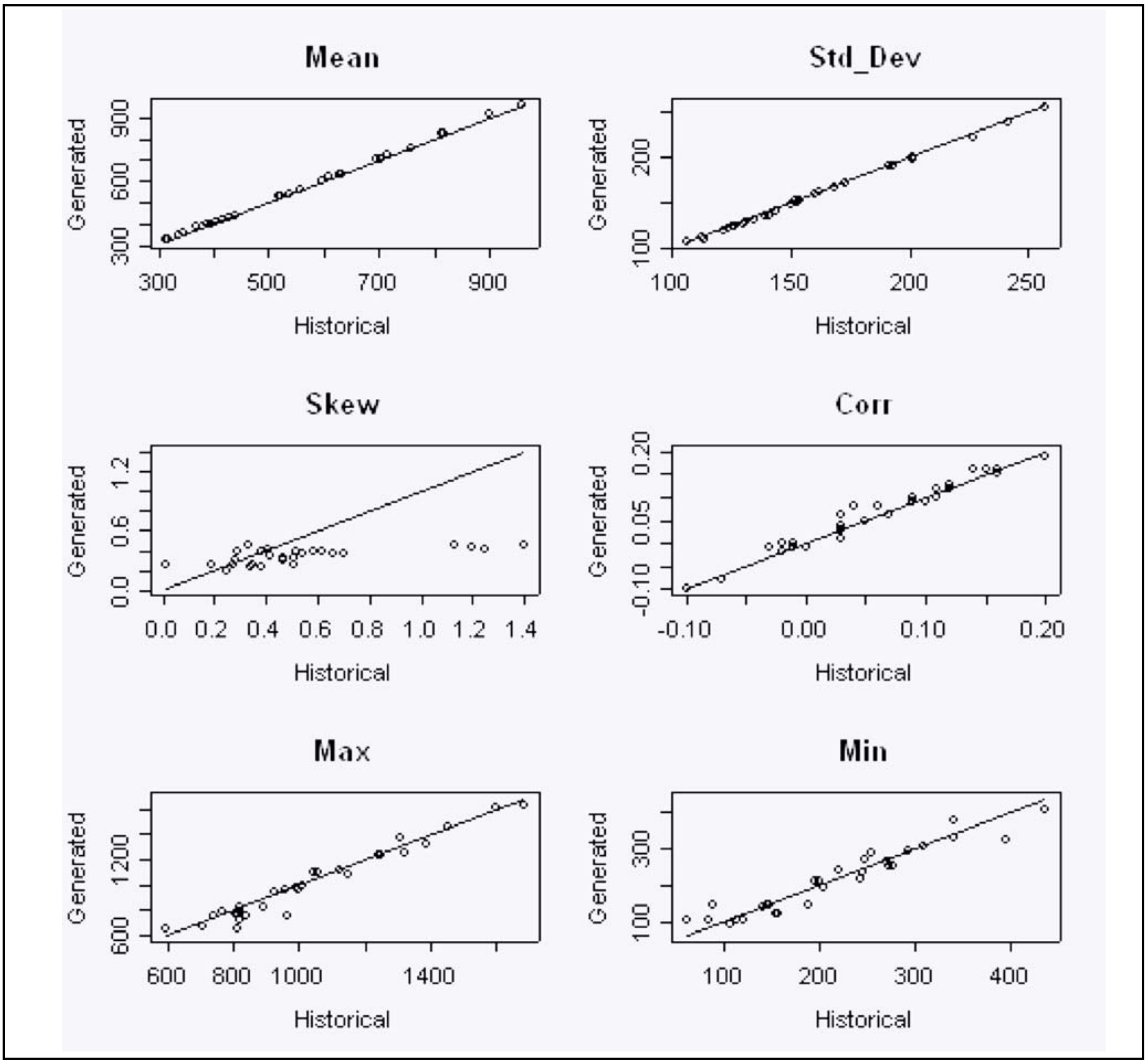

Figure B9. Comparison of Historical and Generated Annual Parameters for the Murrumbidgee Catchment.

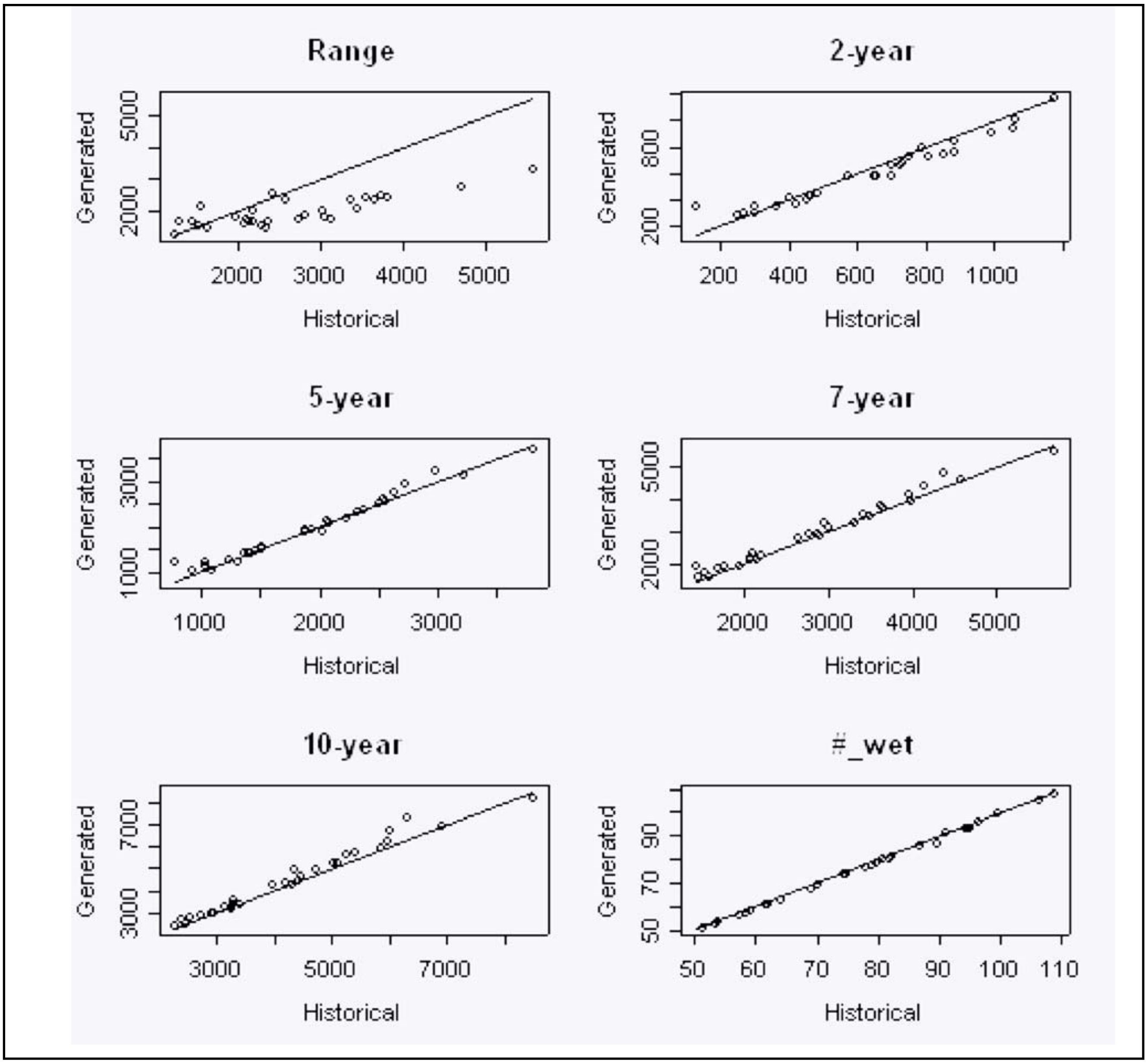

Figure B9. Comparison of Historical and Generated Annual Parameters for the Murrumbidgee Catchment. (Cont.)

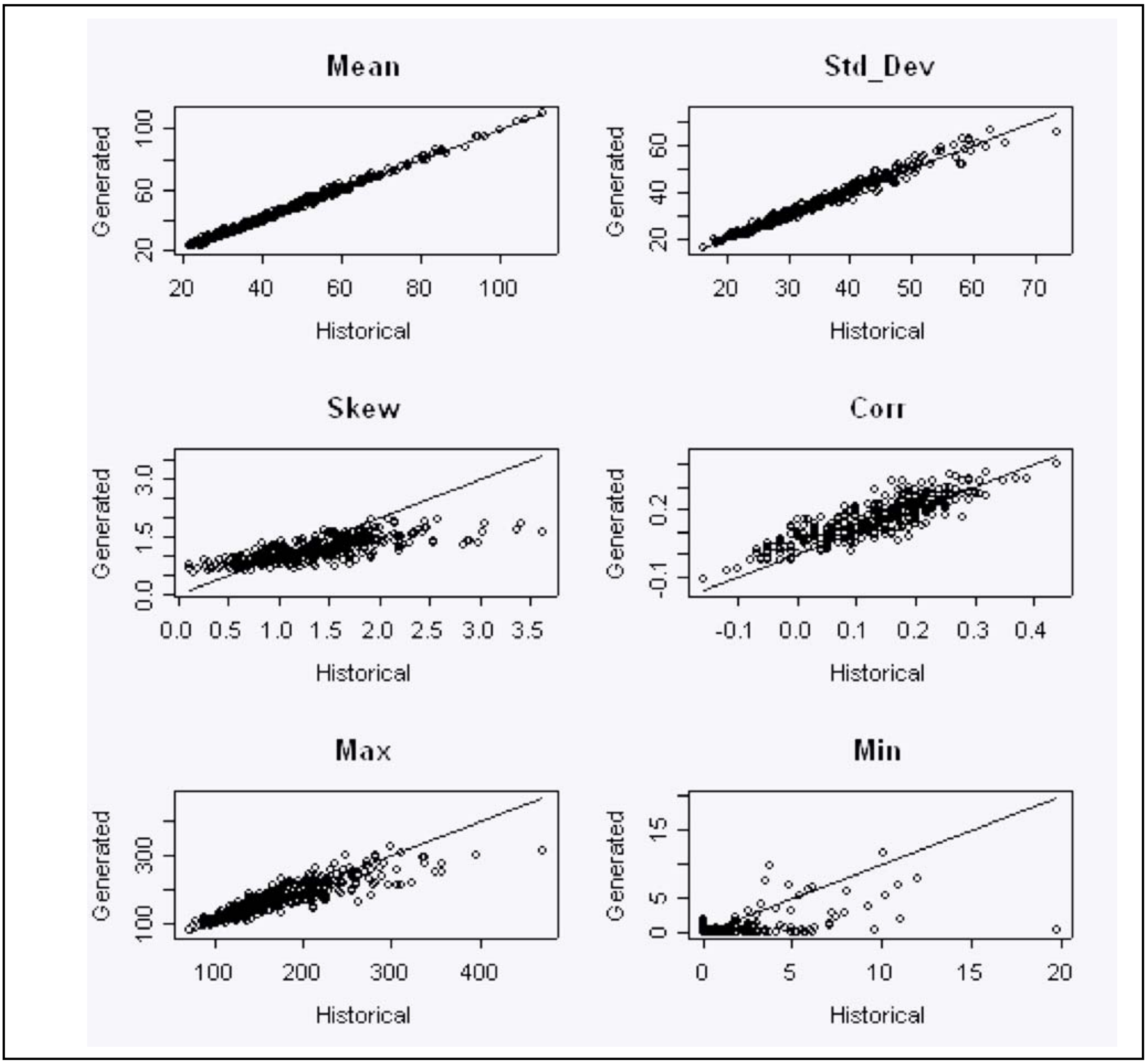

Figure B10. Comparison of Historical and Generated Monthly Parameters for the Murrumbidgee Catchment.

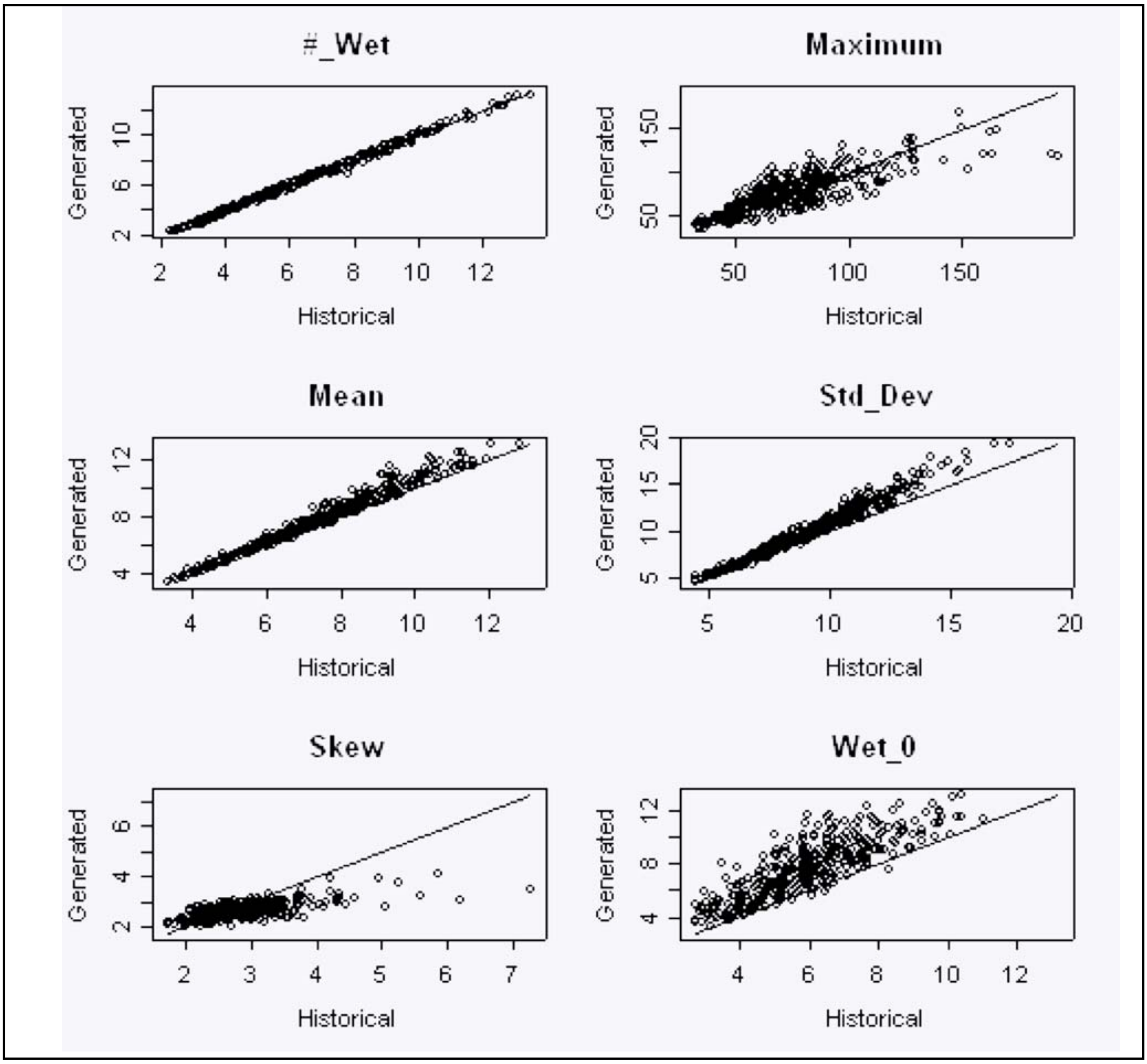

Figure B11. Comparison of Historical and Generated Daily Parameters for the Murrumbidgee Catchment.

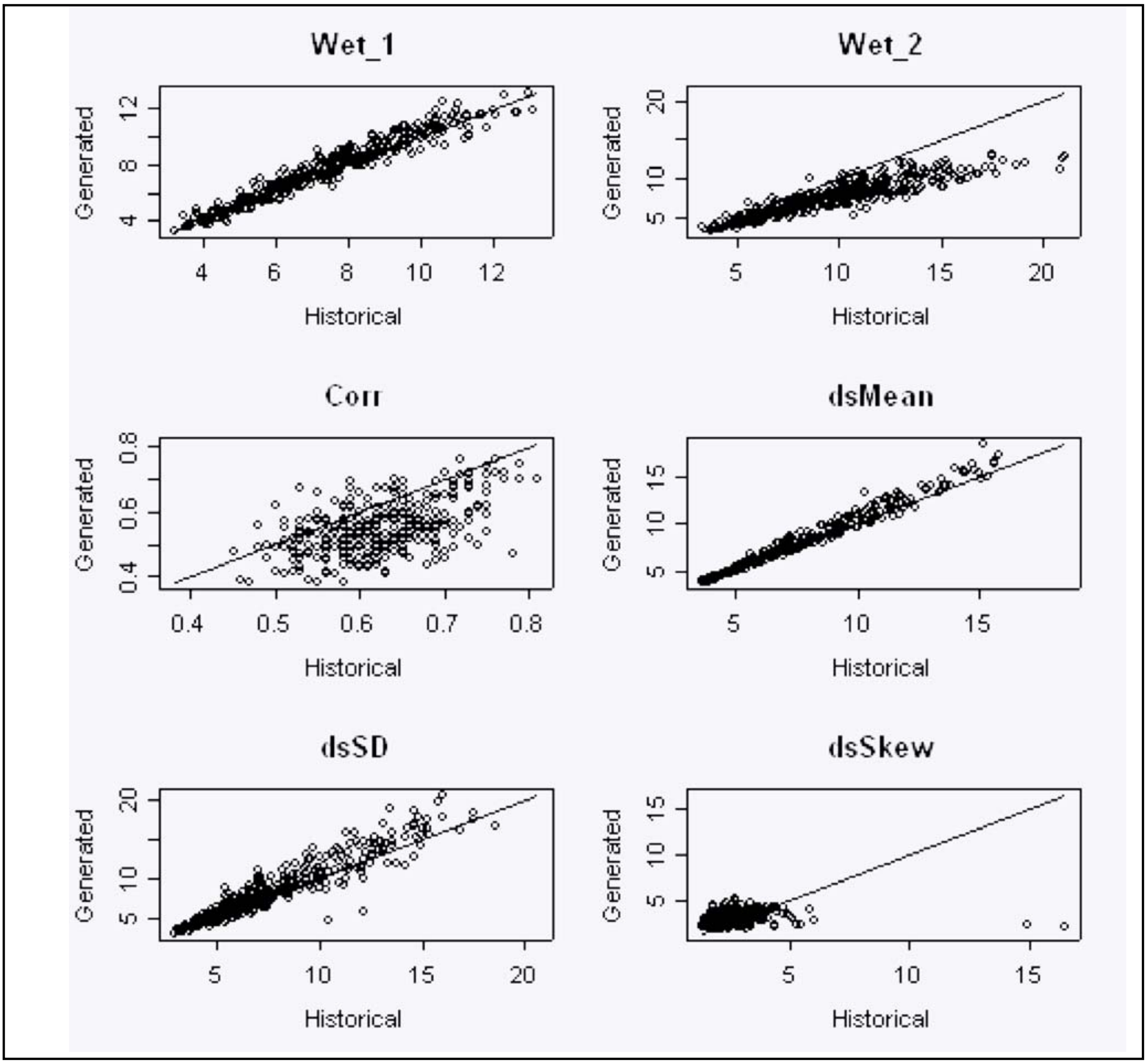

Figure B11. Comparison of Historical and Generated Daily Parameters for the Murrumbidgee Catchment. (Cont.)

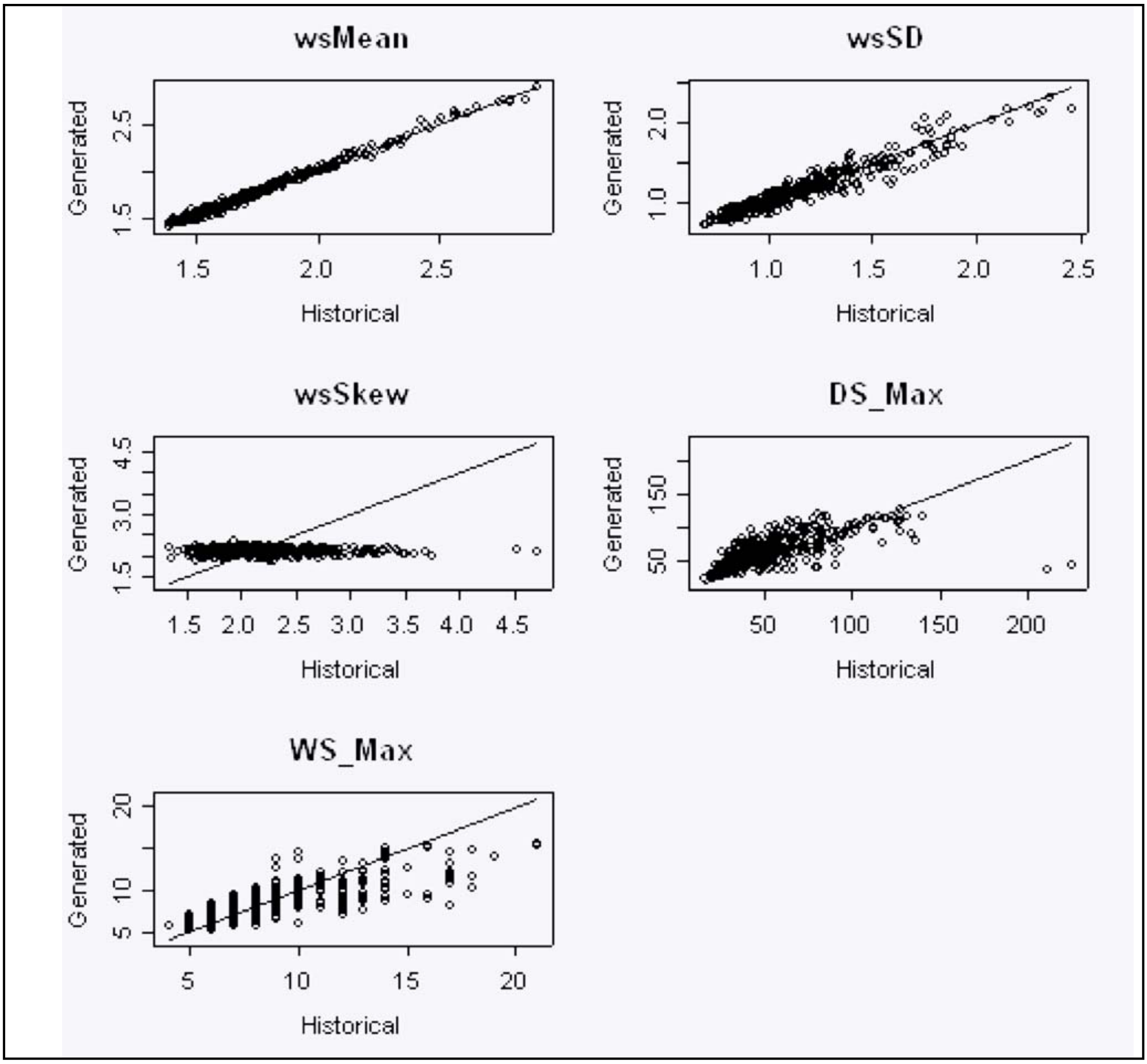

Figure B11. Comparison of Historical and Generated Daily Parameters for the Murrumbidgee Catchment. (Cont.)

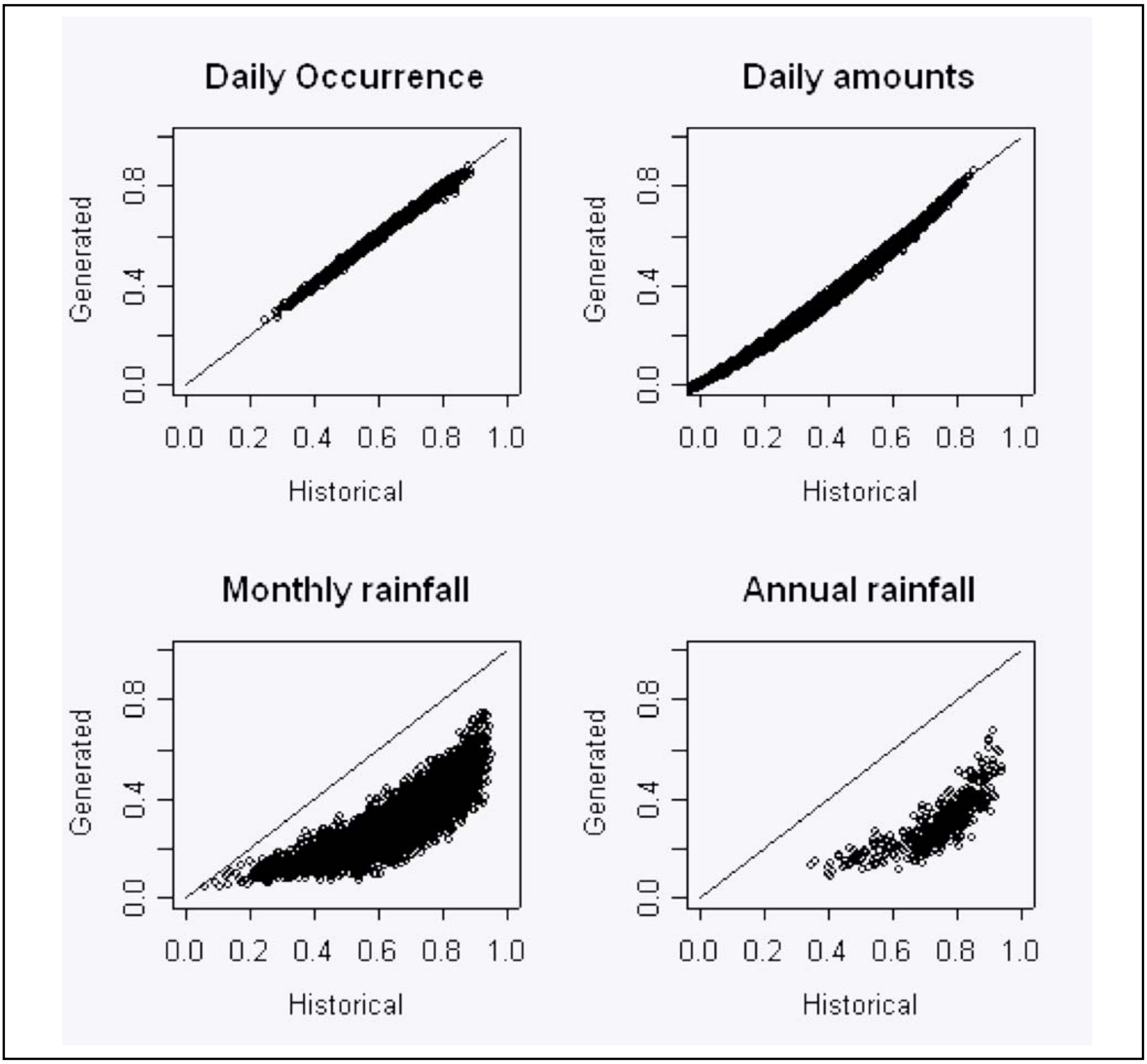

Figure B12. Comparison of Historical and Generated Cross Correlations for the Murrumbidgee Catchment.

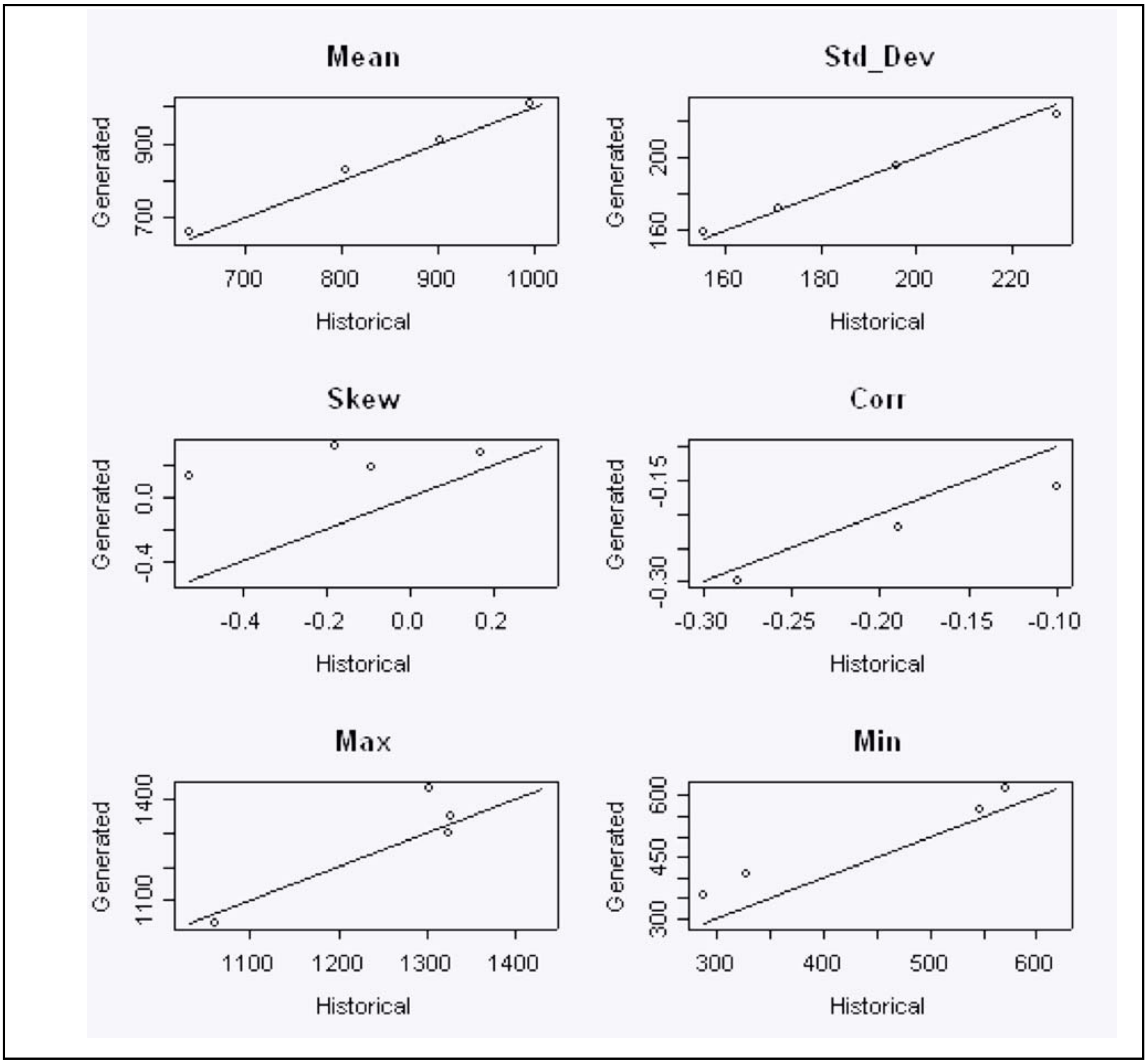

Figure B13. Comparison of Historical and Generated Annual Parameters for the Goulburn-Murray Catchment.

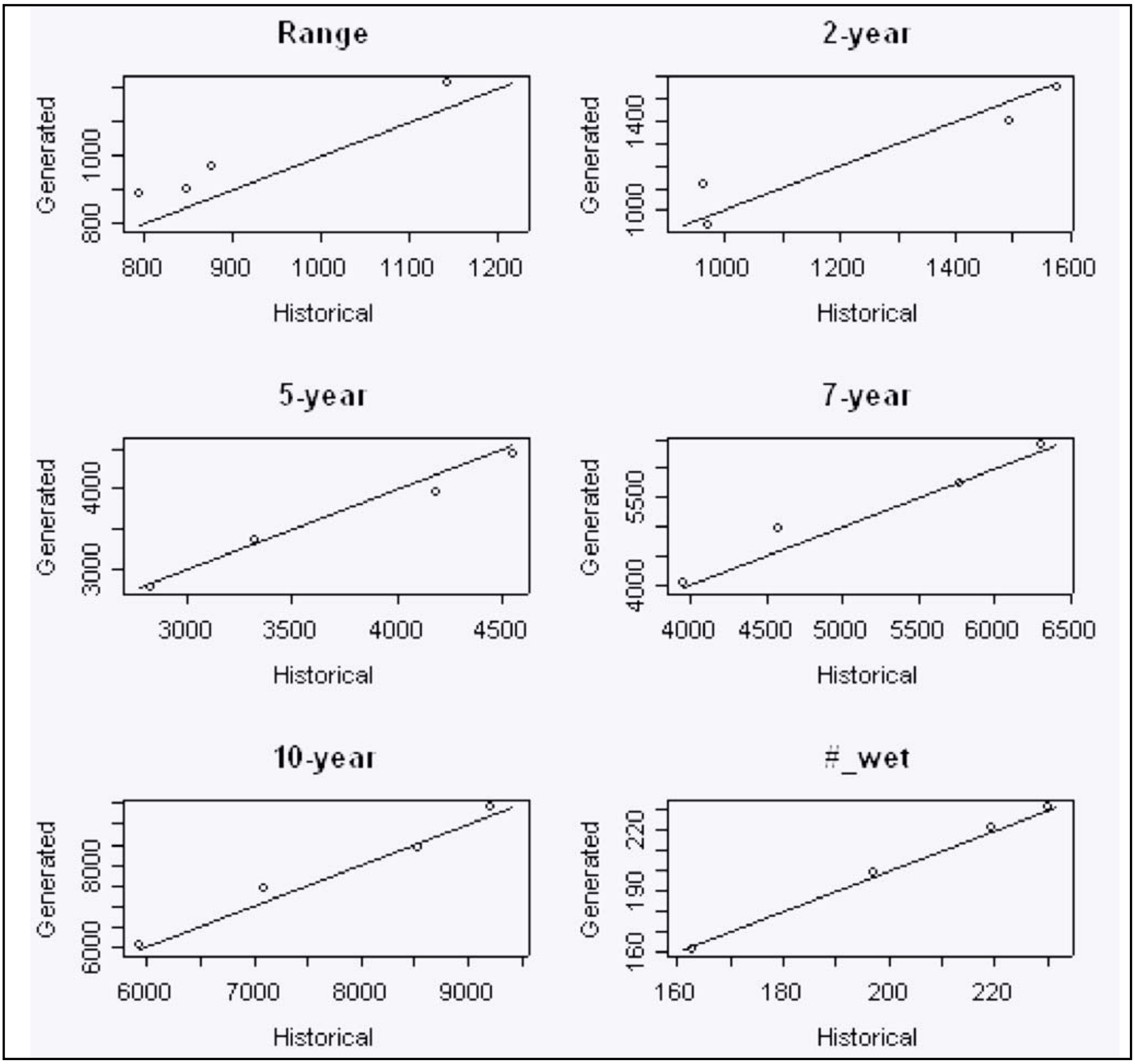

Figure B13. Comparison of Historical and Generated Annual Parameters for the Goulburn-Murray Catchment. (Cont.)

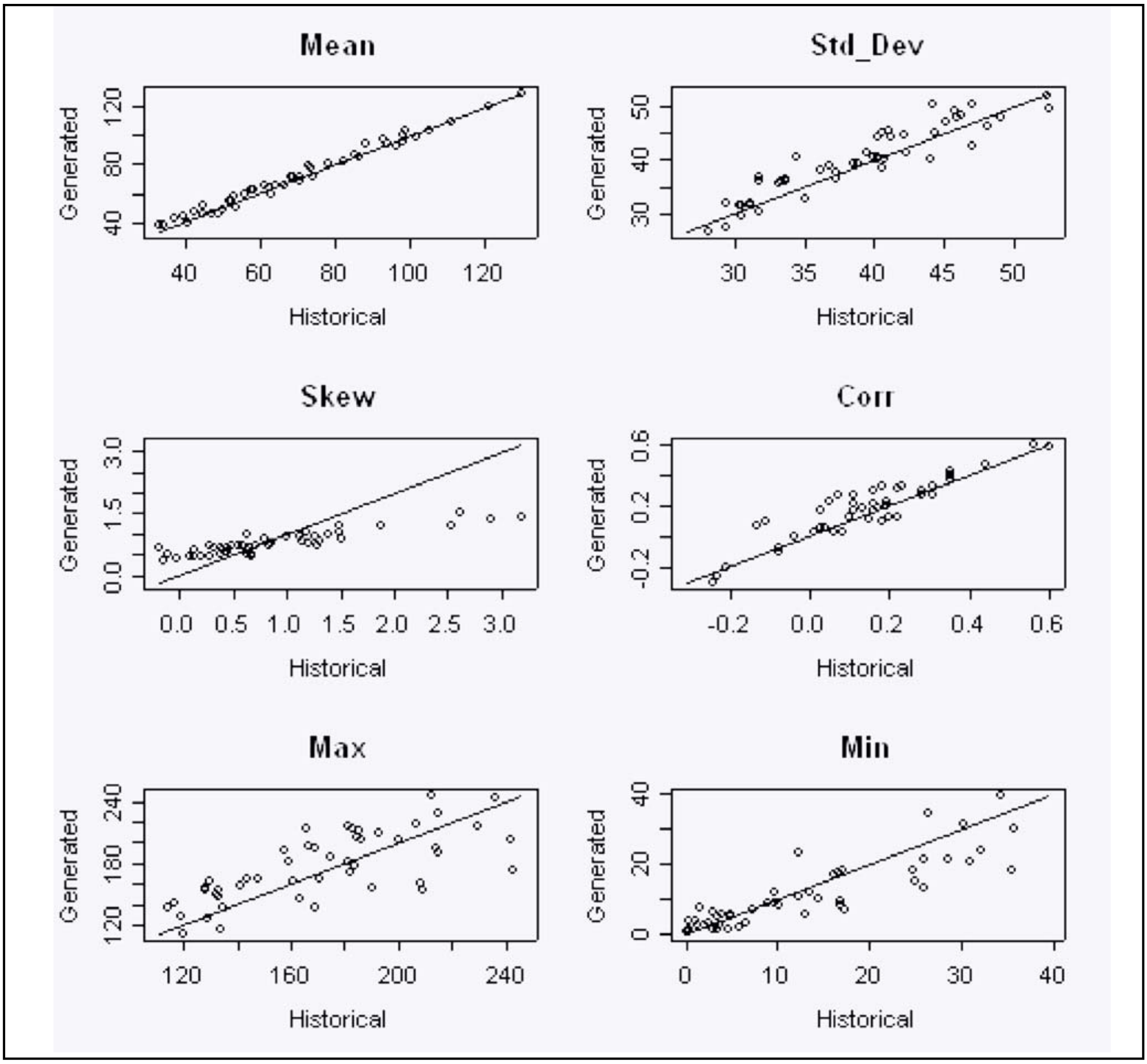

Figure B14. Comparison of Historical and Generated Monthly Parameters for the Goulburn-Murray Catchment.

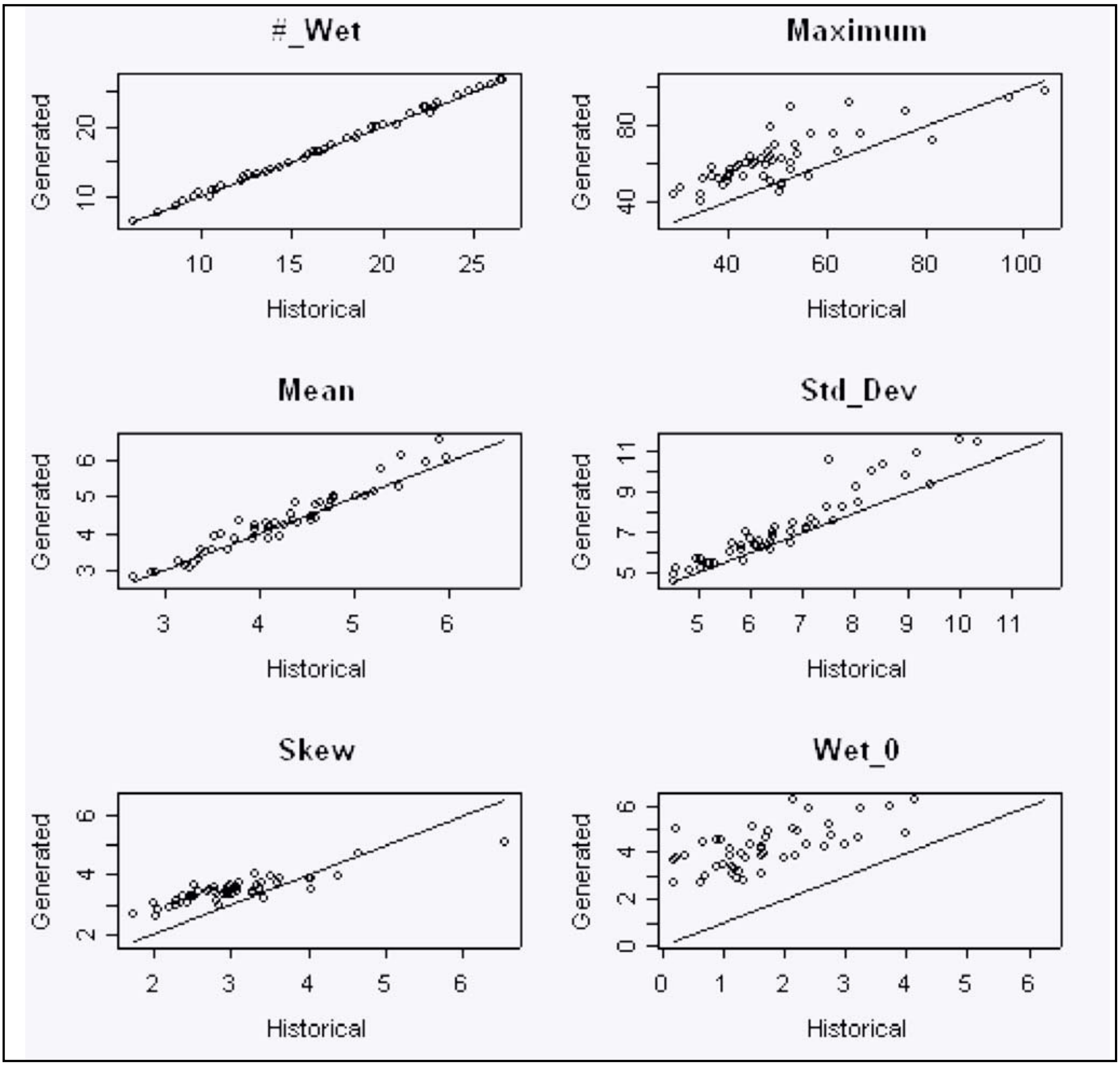

Figure B15. Comparison of Historical and Generated Daily Parameters for the Goulburn-Murray Catchment.

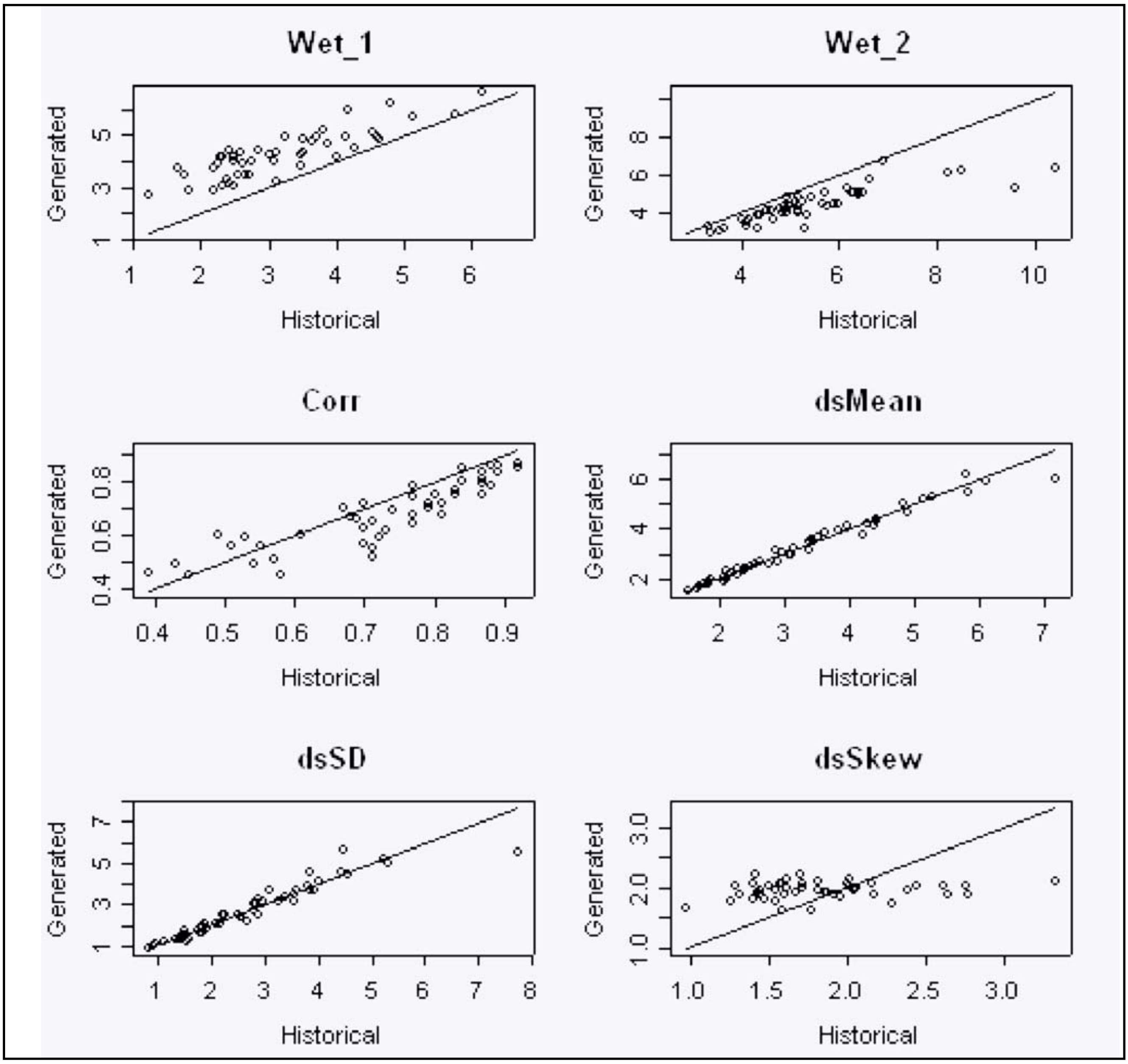

Figure B15. Comparison of Historical and Generated Daily Parameters for the Goulburn-Murray Catchment. (Cont.)

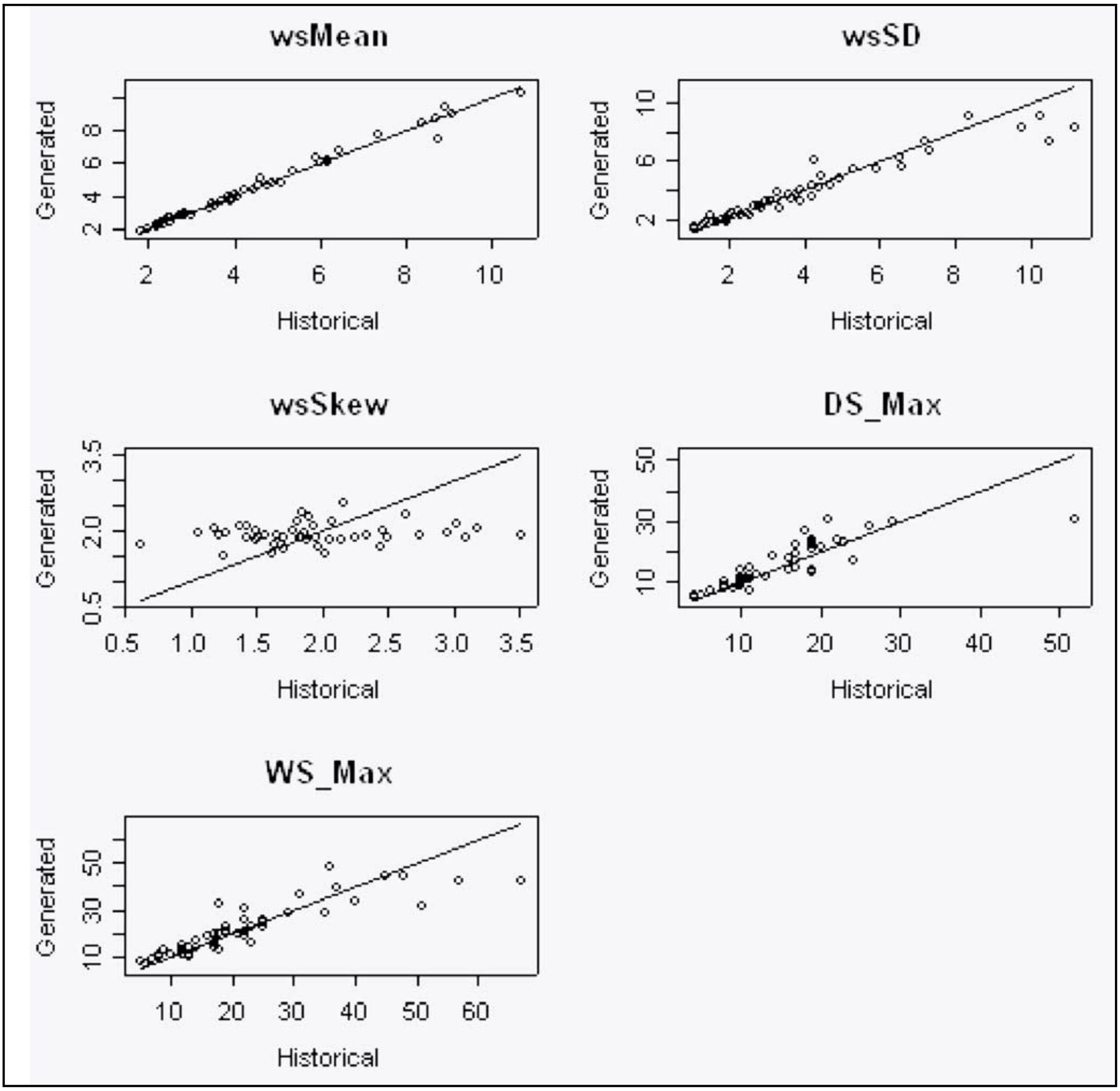

Figure B15. Comparison of Historical and Generated Daily Parameters for the Goulburn-Murray Catchment. (Cont.)

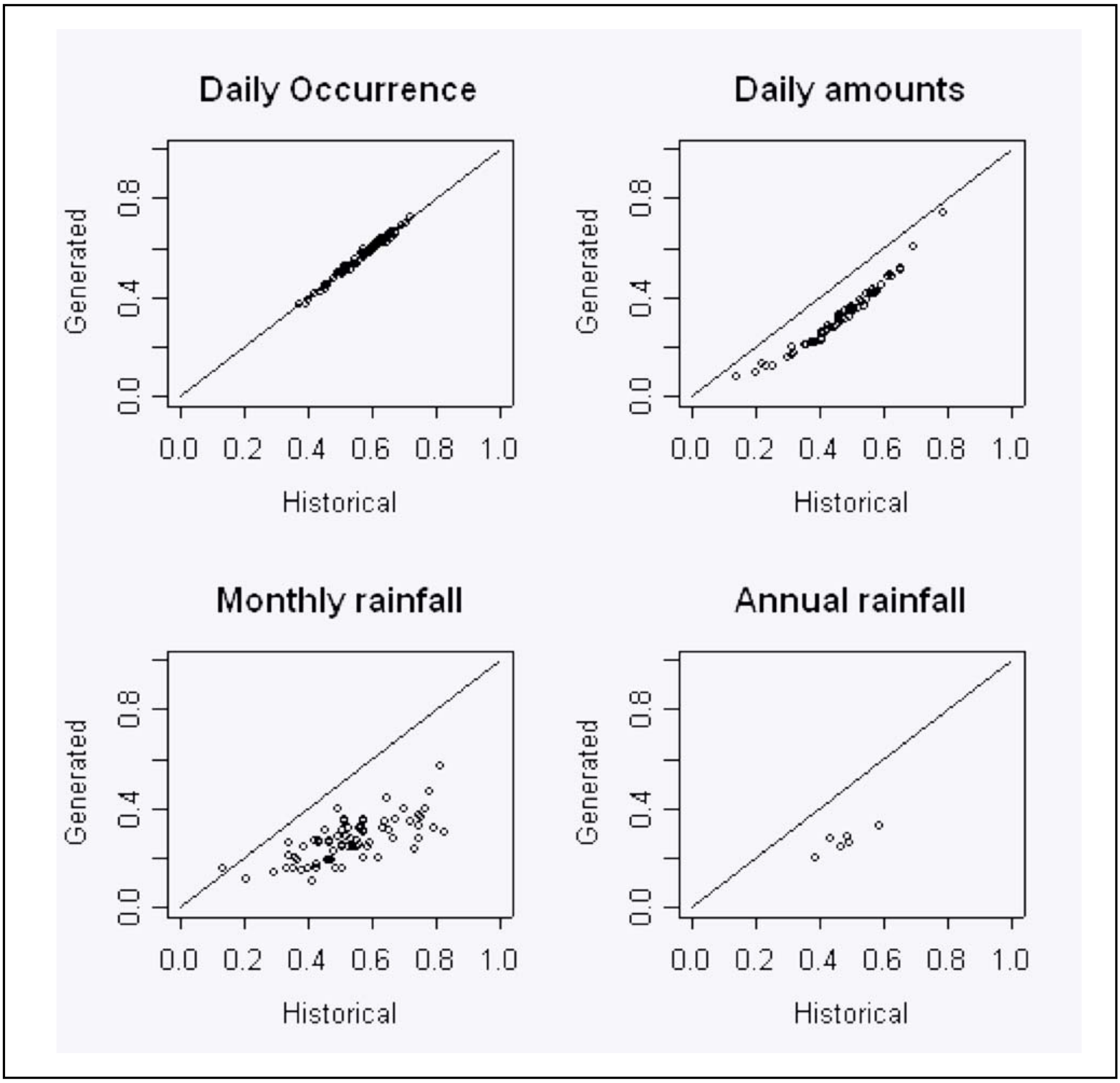

Figure B16. Comparison of Historical and Generated Cross Correlations for the Goulburn-Murray Catchment.

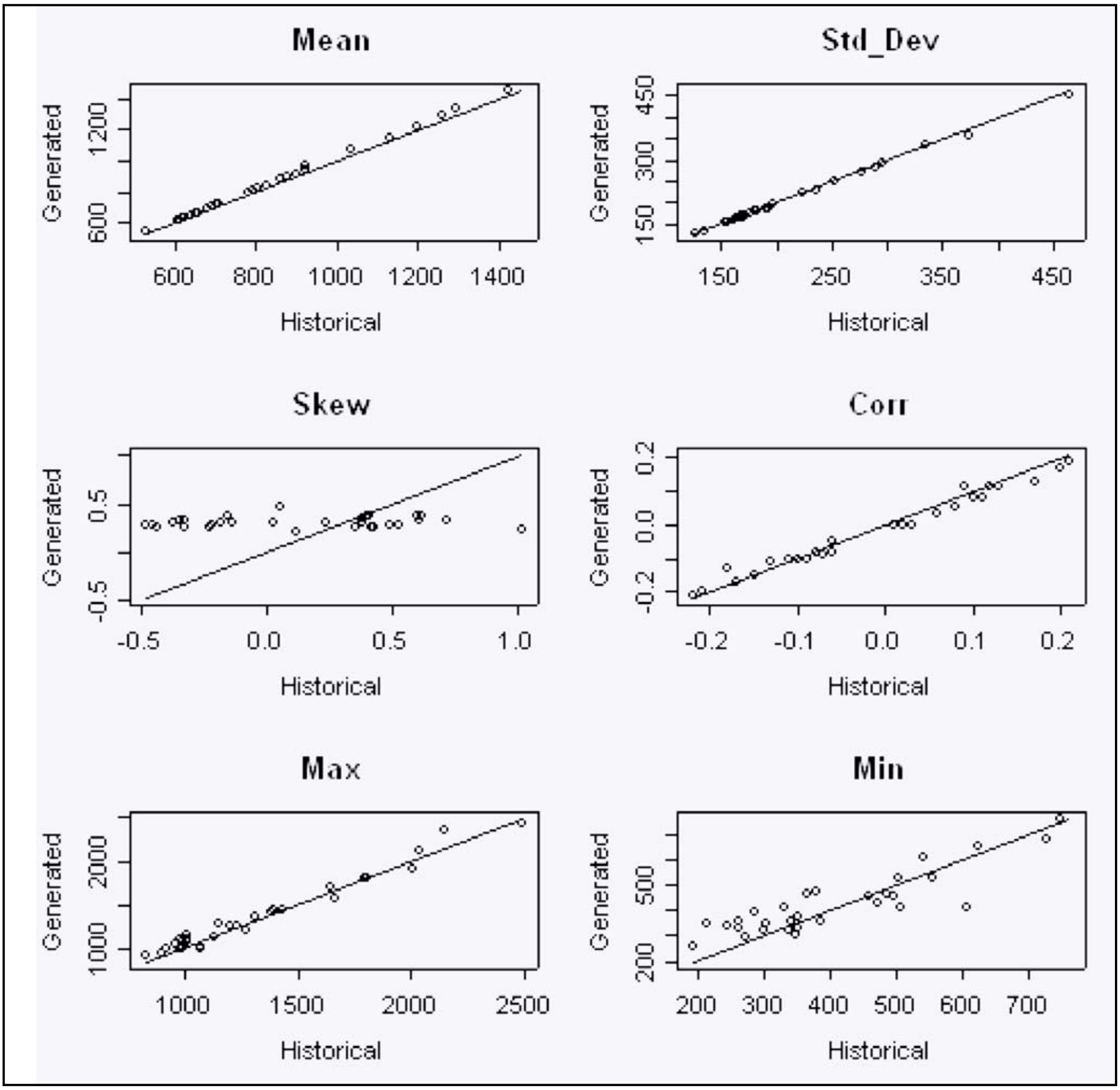

Figure B17. Comparison of Historical and Generated Annual Parameters for the Sydney Region.

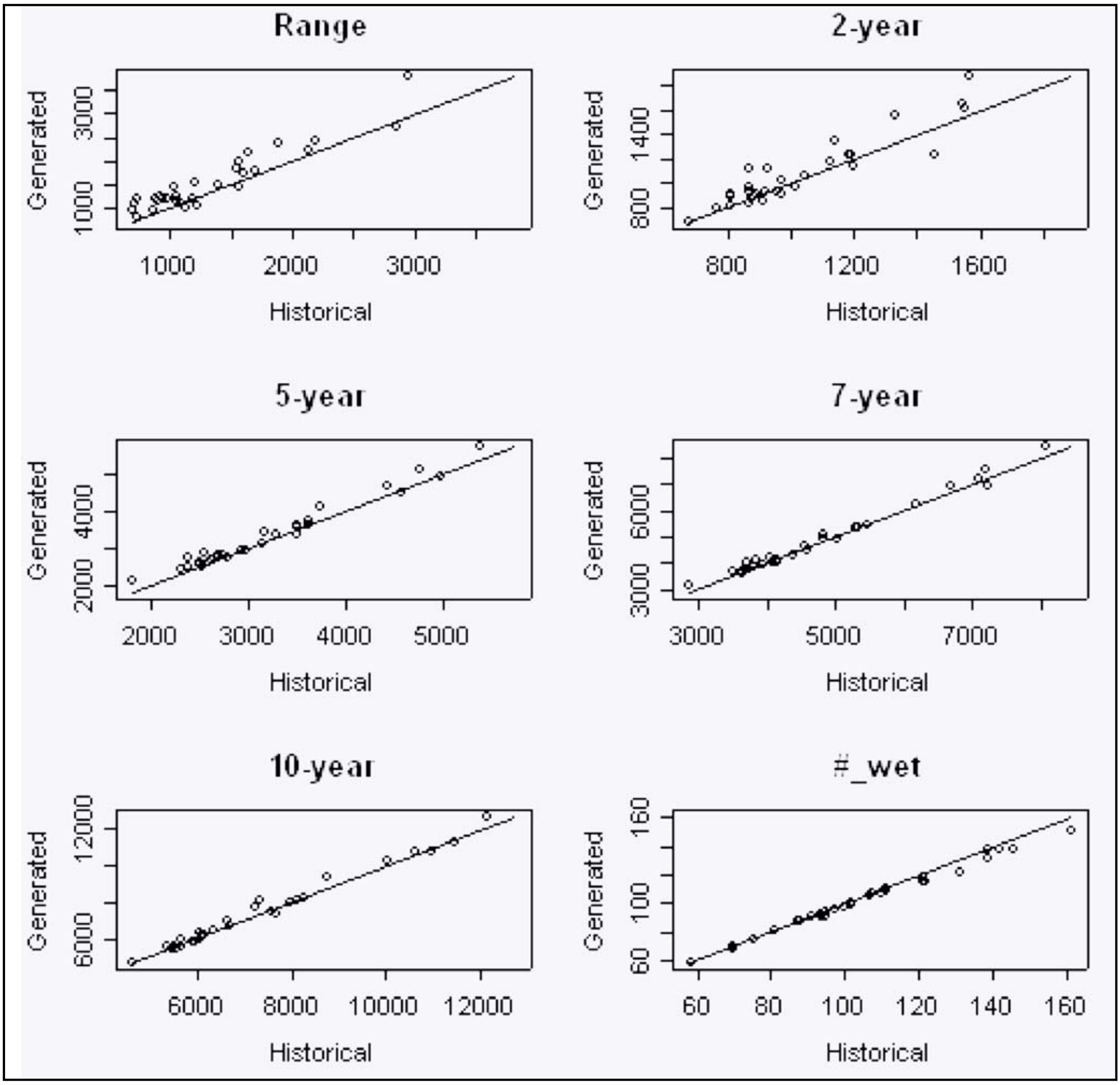

Figure B17. Comparison of Historical and Generated Annual Parameters for the Sydney Region. (Cont.)

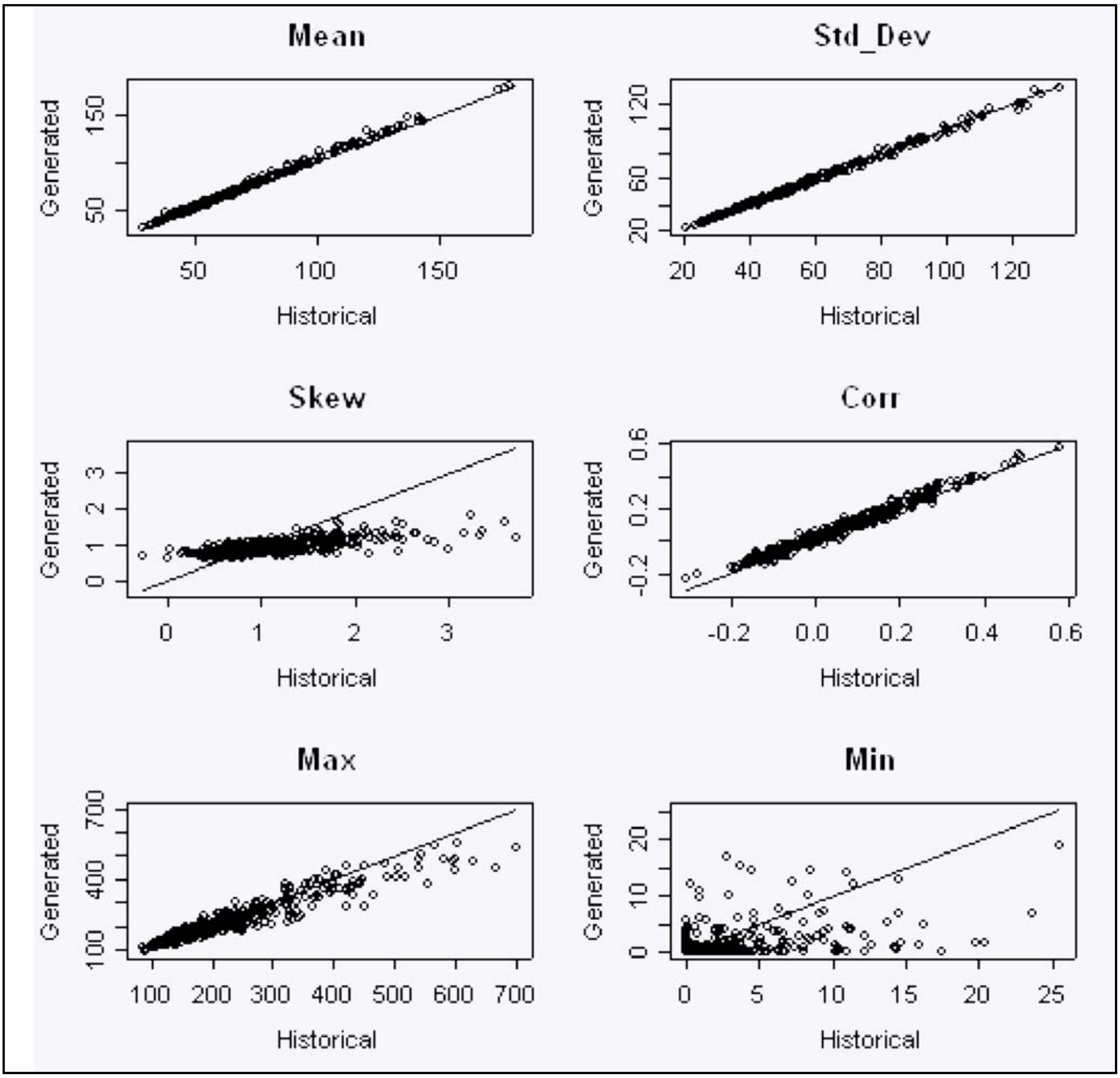

Figure B18. Comparison of Historical and Generated Monthly Parameters for the Sydney Region.

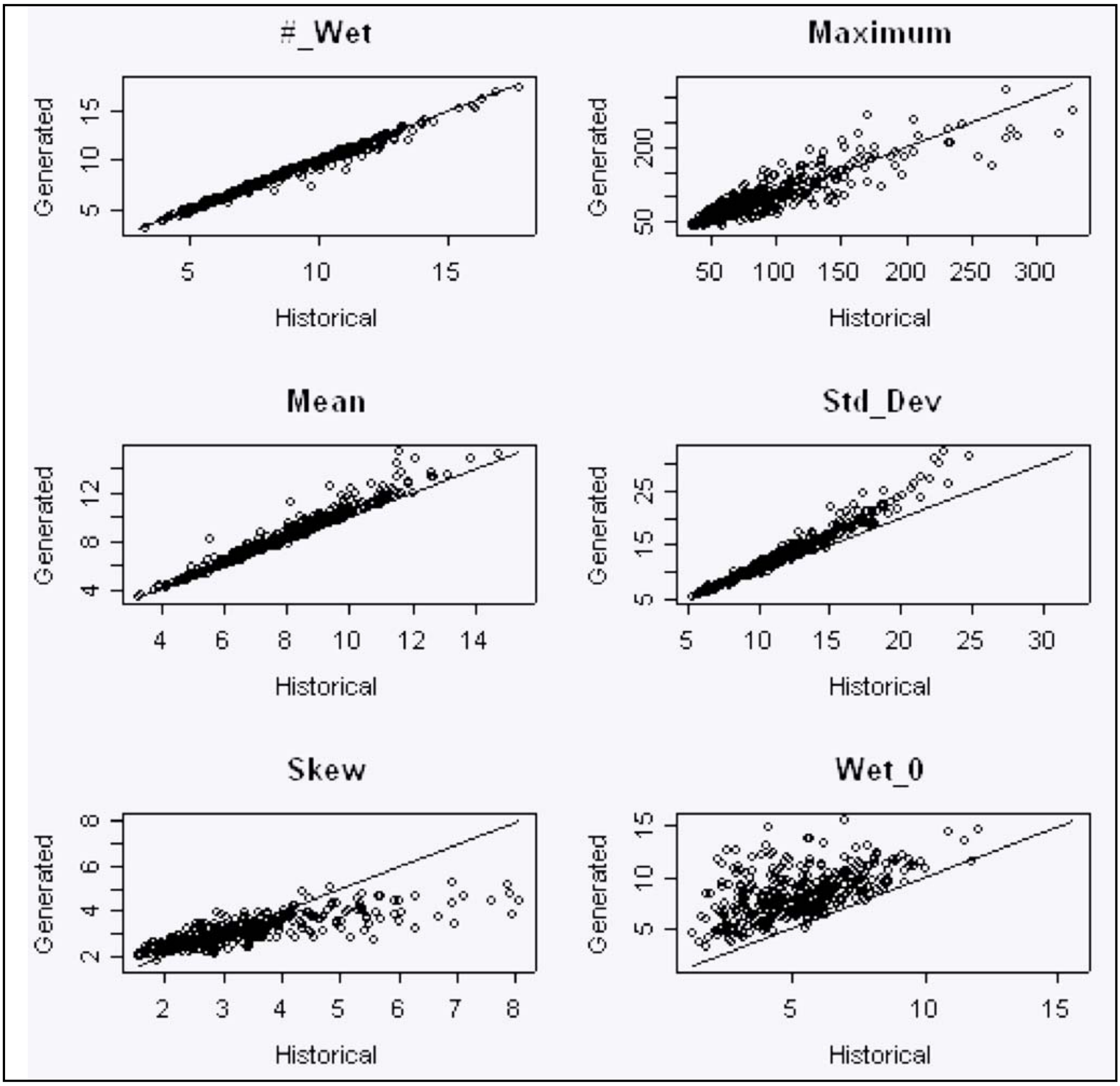

Figure B19. Comparison of Historical and Generated Daily Parameters for the Sydney Region.

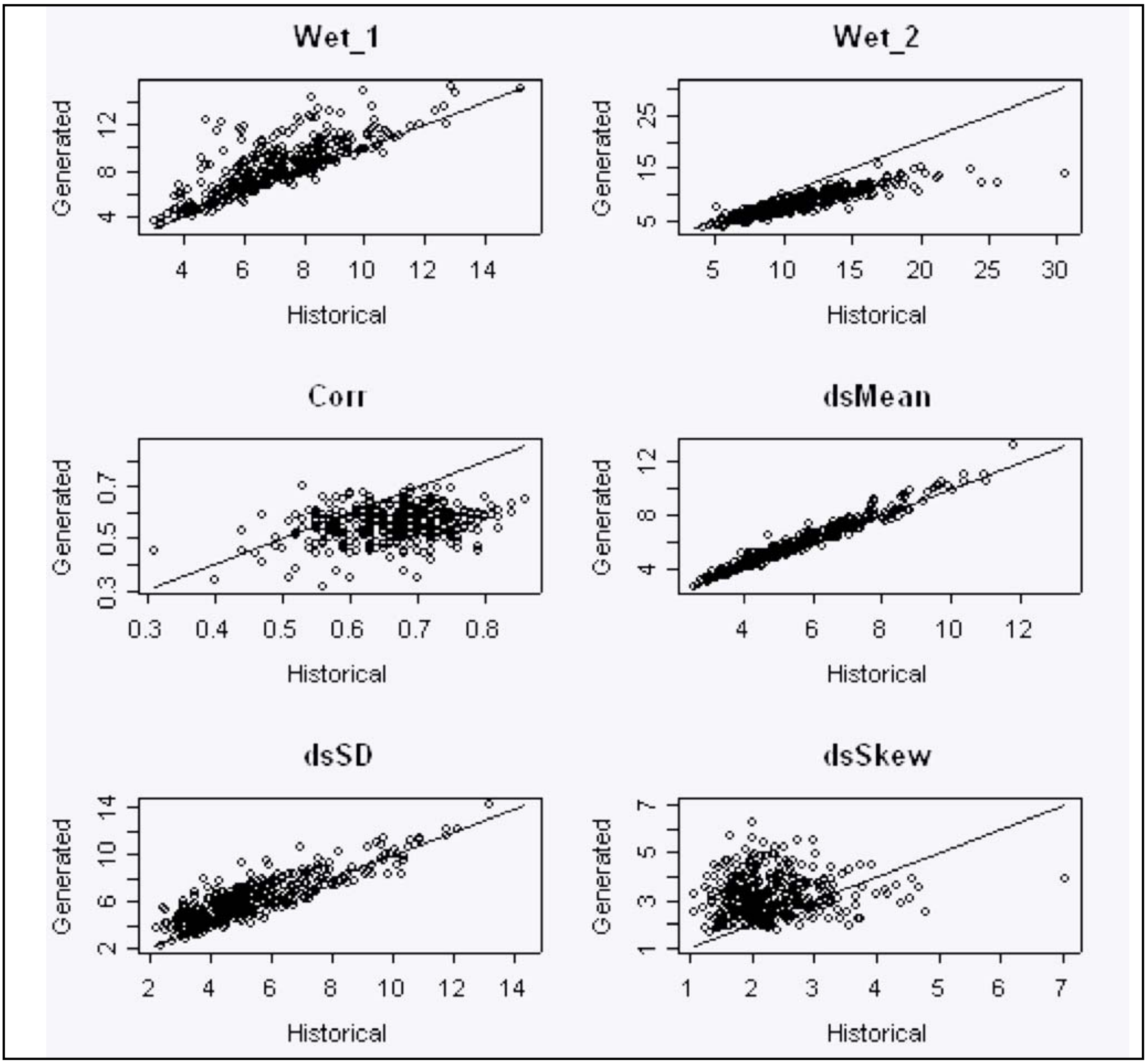

Figure B19. Comparison of Historical and Generated Daily Parameters for the Sydney Region. (Cont.)

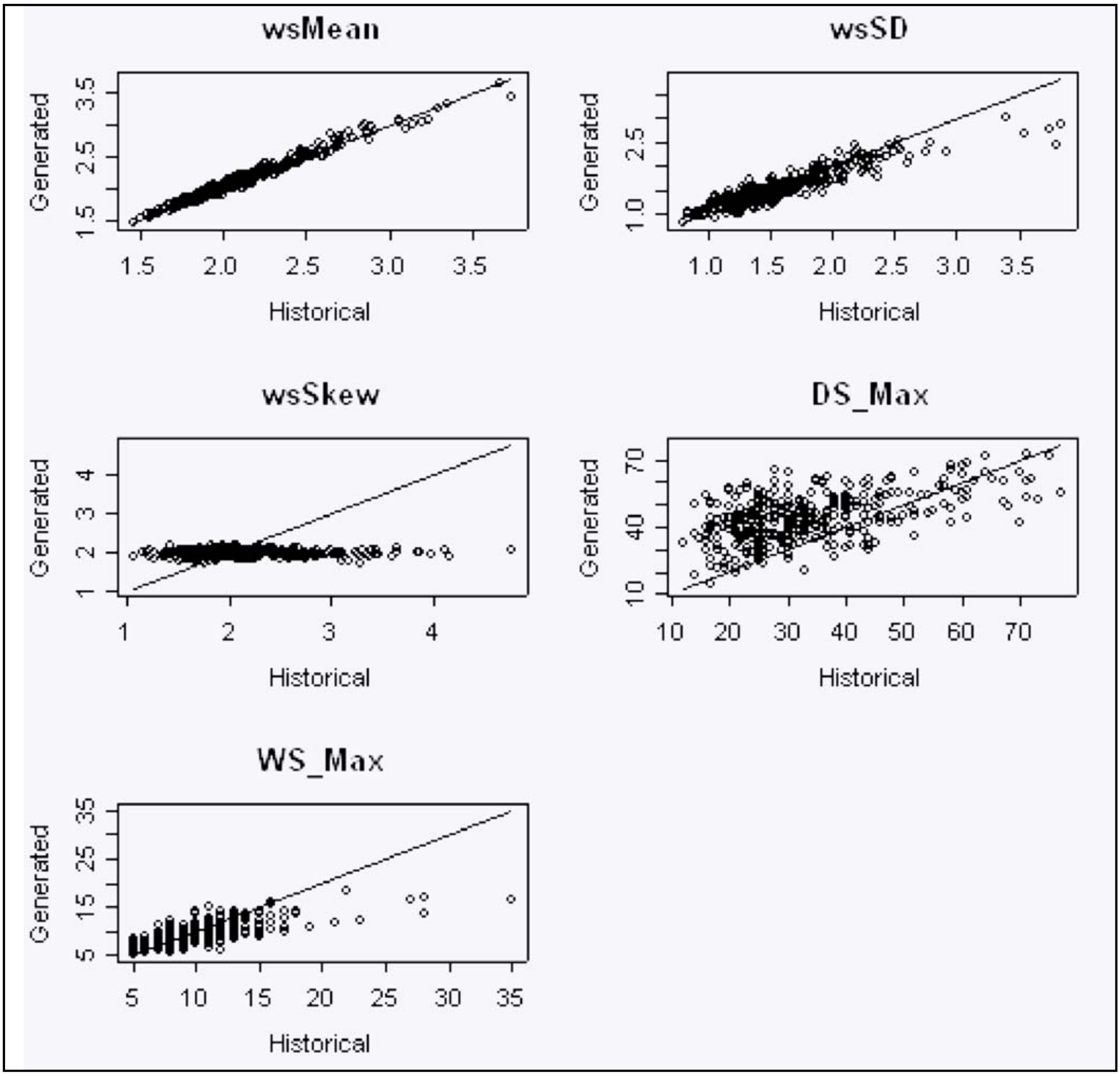

Figure B19. Comparison of Historical and Generated Daily Parameters for the Sydney Region. (Cont.)

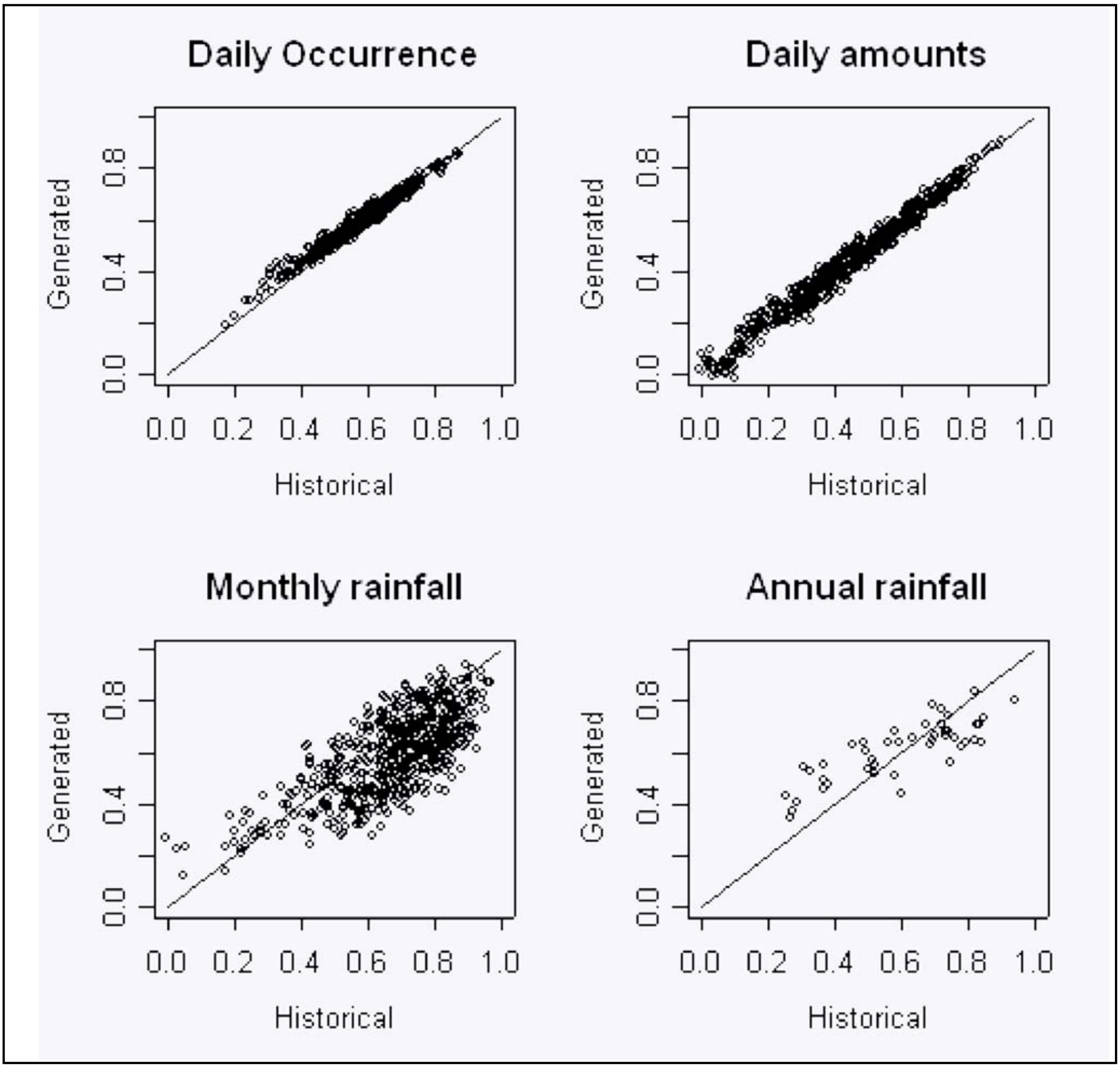

Figure B20. Comparison of Historical and Generated Cross Correlations for the Sydney Region.

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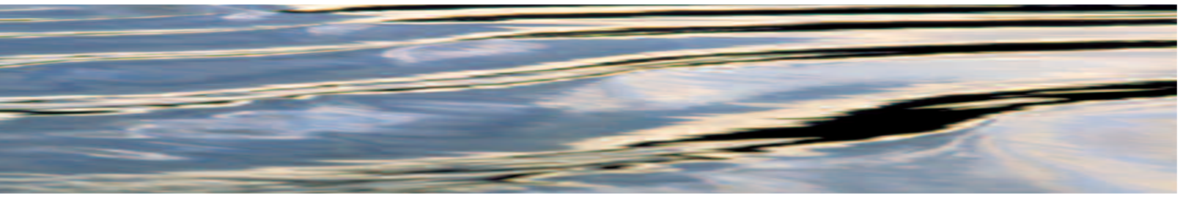

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- Brisbane City Council
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- Department of Sustainability and Environment, Vic
- Goulburn-Murray Water
- Grampians Wimmera Mallee Water
- Griffith University
- Melbourne Water
- Monash University
- Murray-Darling Basin Commission
- Natural Resources and Mines, Qld
- Southern Rural Water
- The University of Melbourne

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- Australian National University
- National Institute of Water and Atmospheric Research, New Zealand
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